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FLOW CHARACTERISTICS IN ARTERIAL JUNCTIONS

BY



CHRISTIAN L. ROUSSEL

A THESIS

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ABSTRACT

The manner in which the steady flow divides at an arterial junction was experimentally established in terms of the significant dimensionless parameters: the entrance Reynolds number, Re , the ratio, β , of the side branch diameter to the main branch diameter and the non-dimensional angle of branching, α .

The type of bifurcation examined was the "side branch", where a single branch leaves the parent trunk at an angle dependent on the anatomical position.

The ratio, γ , of the flow rate in the side branch to the flow rate in the main branch was found to increase with a decrease of the Reynolds number and the angle of branching and with an increase of the ratio of diameters, β .

A visualization technique proved the existence of two interdependent separation regions, one in each branch, responsible for the variation of the mass flow ratio as function of Re, β, θ . The formation, growth and shedding of vortices in the separated region of the main branch and the double-helicoidal flow in the side branch have been observed.



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LIST OF SYMBOLS

a	$= \text{Re}(Q_d + Q_D)^{-1}$, in (liters per minute) $^{-1}$
b	$= \text{Re}(Q_d + Q_D)^{-1}$, in (U.S. Gallons per minute) $^{-1}$
c	fluid specific heat per unit mass
Cu	copper atom
$\text{Cu}(\text{OH})_2$	copper hydroxide molecule
d	side-branch internal diameter
D	mainline-branch internal diameter
e	fluid specific internal energy
$\bar{e}$	electron
E	$= U^2/c\Delta T$, Eckert number
Eu	$= \Delta P/\rho U^2$, Euler number
$\vec{F}$	local body force
H^+	hydrogen ion
H_2	hydrogen molecule
H_2O	water molecule
k	fluid thermal conductivity
L	entrance length
Le	$= L/R$, length ratio
O_2	oxygen molecule
OH^-	hydroxyl ion
p	local static pressure

ΔP	maximum pressure drop in the entrance region
$Pé$	$= \rho c UR/k$, Péclet number
q	heat generated per unit mass
$\vec{q}$	local fluid velocity vector
Q_d	flow rate of the side branch
Q_D	flow rate of the main branch
r	radial space coordinate
r_1	radius of curvature (see Figure 3.1)
r_2	radius of curvature (see Figure 3.1)
R	$= D/2$, tube internal radius
Re	$= u'D/\nu$, Reynolds number
t	time variable
T	local temperature
ΔT	maximum temperature difference in the entrance region
u	longitudinal velocity component
u'	average longitudinal velocity at the bifurcation entrance
U	maximum longitudinal velocity at the bifurcation entrance
v	radial velocity component
V	reference radial velocity
x	longitudinal space coordinate
α	$= \theta/90^\circ$, non-dimensional angle of branching
β	$= d/D$, branch diameter ratio
γ	$= Q_d/Q_D$, mass flow ratio
ϕ	dissipation function per unit mass

θ	angle of branching
λ	$= U/u'$, ratio of maximum to mean longitudinal velocity at the bifurcation entrance
ν	fluid kinematic viscosity
ρ	fluid density
σ	$= 4Le/Re$, dimensionless entrance length
τ_d	thickness of the separated region in the side branch (see Figure 4.22)
τ_D	thickness of the separated region in the main branch (see Figure 4.22)

CHAPTER I

INTRODUCTION

1.1 Introductory Remarks

As the human knowledge increases, one tries to apply it to a better understanding of oneself. This is particularly true in medical and paramedical research for which curing diseases and saving lives are the two main goals.

Since diseases and defects of the human cardiovascular system remain an important cause of death, researchers are expending considerable energy to understand this complex system. The flow of blood through the vessels is one of the aspects of this system well suited for the engineer to investigate, due to the fact that it is essentially a problem in fluid mechanics, although a rather complicated one.

Because of the high degree of complexity of the circulatory system, a systematic application of the physical laws and equations valid in viscous flow would be a considerable task, indeed. However, the use of experimental data as background and justification of theories based on some simplifications of the physiological model, can lead to important results and will contribute to the extension of human knowledge.

1.2 Literature Survey

One way of studying a highly complicated problem is to simplify it as far as possible, to study the new problem and then add more and more complexity until the study of the real problem can be done. For that reason, the blood flow study is very much indebted to the French physician Poiseuille who, for his blood flow study, turned to the study of fully-developed steady flows of Newtonian fluids in straight rigid tubes.

The fully-developed flows appear when the effects of the ends of the tube are negligible. H.L. Langhaar¹ and H.B. Atabek² considered these end effects in developing an inlet length theory for steady and periodic flows in straight rigid tubes, respectively. J.H. Gerrard³ experimentally investigated pulsating turbulent flow in a tube.

N.R. Kuchar and S. Ostrach⁴ studied entrance effects associated with laminar flows of viscous fluids in circular elastic tubes. Later, in 1966, B.H. Ruterbories and S. Ostrach⁵ measured velocity profiles and pressure distributions of pulsating flow in a flexible tube.

The problem becomes even more complex in the case of branched vessels and, consequently, most of the work in this area is done with the help of experimental models.

The study of steady flows through branches is of interest not only to the blood flow research but also in air-conditioning or air flows through the lungs. This depends on the type of bifurcation

and the kind of results required. For example, A. Vazsonyi⁶ gave formulas permitting calculation of losses in duct branches, S.F. Gilman⁷ studied the pressure losses of dividing flow fittings and T.J. Pedley, R.C. Schroter and M.F. Sudlow⁸ studied the flow and the pressure drop in systems of repeatedly branching tubes. They used Y-type bifurcations as experimental representation of the lungs airways, obtained velocity profiles and energy dissipations through these bifurcations.

When one wants to study pulsatile flow through a bifurcation, a new problem appears due to pulse wave reflections. One of the earlier attempts to analyse this phenomenon was done by R.L. Evans et al.⁹ who compared reflections in a model with those in an animal. In 1966 J.D. Martin and M.E. Clark¹⁰ experimentally and theoretically analysed wave reflections in branched flexible conduits. R.H. Cox¹¹ and G.S. Malindzak¹² made more sophisticated theoretical and in-vivo analyses of the wave reflections, respectively.

The influence of the branching on the critical Reynolds number has been determined by L.J. Krovetz¹³, who found a lower critical Reynolds number in branched tubes than in straight tubes. The hydrodynamic stability appeared independent of the varying angles of branching.

The occurrence and the severity of the separated flow, as a function of the flow rate, have been studied by D.J. Schneck and W.H. Gutstein¹⁴. They noted a direct correlation between the occurrence of arteriosclerotic plaques and the angle of bending and branching. The correlation between turbulent flow, thrombosis and arterial lesions

has been also pointed out by J.R. Mitchell and C.J. Schwartz¹⁵.

A more complete investigation of pulsatile flow in a side-branch bifurcation has been done by C.M. Rodkiewicz and D.H. Howell¹⁶, who systematically studied the manner in which the flow divides at an arterial bifurcation as a function of the significant dimensionless parameters: Reynolds number, unsteadiness parameter and velocity fluctuation parameter. They found that for certain values of these parameters, more flow goes into the side branch. This leads to the important conclusion, that the heart, activated by an impulse from the brain, can supply more blood to the preferential area of the human body, by changing the values of these three parameters.

1.3 Objectives of the Thesis

In order to have a better understanding of the flow through a side-branch bifurcation and find why and when the mass flow ratio γ is greater than one, it has been decided, from the results obtained by C.M. Rodkiewicz and D.H. Howell¹⁶, to study the steady-flow characteristics in an arterial junction. The parameters for this study will be:

- the entrance Reynolds number,
- the geometry of the bifurcation,
- the smoothness of the inside edges of the bifurcation.

Qualitative as well as quantitative results will be produced giving the variation of γ , the occurrence and the severity of the separated flow and the location of the separation point, as functions of the total flow rate.

CHAPTER II

DERIVATION OF THE PARAMETERS FROM A

MATHEMATICAL APPROACH

2.1 Introductory Remarks

As previously explained, this thesis is mainly concerned with the in-vitro study of the blood flow characteristics in arterial junctions.

The present study is limited to the steady case of a homogeneous incompressible Newtonian fluid of constant viscosity flowing through a side-branch bifurcation.

2.2 Governing Equations

2.2.1 The General Continuity Equation is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0 \quad (2.1)$$

in which ρ is the fluid density and $\vec{q}$ the fluid velocity at a point. Applying the conditions of steadiness ($\partial/\partial t \equiv 0$) and incompressibility ($\rho = \text{constant}$), the equation reduces to:

$$\nabla \cdot \vec{q} = 0 \quad (2.2)$$

2.2.2 The General Navier-Stokes Equations for a fluid with constant viscosity can be expressed as follows:

$$\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho} \nabla p + \frac{1}{3} \nu \nabla (\nabla \cdot \vec{q}) + \nu \nabla^2 \vec{q} \quad (2.3)$$

where $\vec{F}$ and p represent the body force and the static pressure at the point under consideration and ν is the fluid kinematic viscosity. Writing these equations for the steady flow of an incompressible fluid, we finally obtain, using equation (2.2) and neglecting the body force ($\vec{F} = \vec{0}$):

$$(\vec{q} \cdot \nabla) \cdot \vec{q} = - \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{q} \quad (2.4)$$

2.2.3 The Energy Equation can be written, for the sake of completeness of this chapter, in its general form as follows:

$$\frac{De}{Dt} = q - \frac{1}{\rho} p \nabla \cdot \vec{q} + \frac{1}{\rho} \nabla (k \nabla T) + \nu \phi \quad (2.5)$$

in which e is the specific internal energy, q the heat generated per unit mass, k the fluid thermal conductivity, ϕ the dissipation function per unit mass and T the local temperature.

In the case of the steady flow of an incompressible fluid with constant properties and without heat generation, equation (2.5) becomes:

$$c(\vec{q} \cdot \nabla) T = \frac{k}{\rho} \nabla^2 T + v\phi \quad (2.6)$$

2.3 Derivation of the Parameters

2.3.1 Introductory Remarks

The purpose of this section is to derive the parameters describing the flow entering the bifurcation; indeed, the flow through the bifurcation depends on these parameters. This necessitates further simplifications of the physiological model:

1. Upstream of the bifurcation, the vessel is considered to be straight and of constant circular cross-section and of rigid walls.
2. The system is assumed to be dynamically and geometrically axisymmetric.

2.3.2 Reduction of the Governing Equations

The cylindrical polar coordinate system, as indicated in Figure 2.1, is the best one to be used in the case of a flow through a straight pipe and can also be used to describe the flow through the bifurcation.

Written in this coordinate system, the governing equations (2.2), (2.4) and (2.6) become, taking into consideration the axial symmetry upstream of the bifurcation:

1. Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \quad (2.7)$$

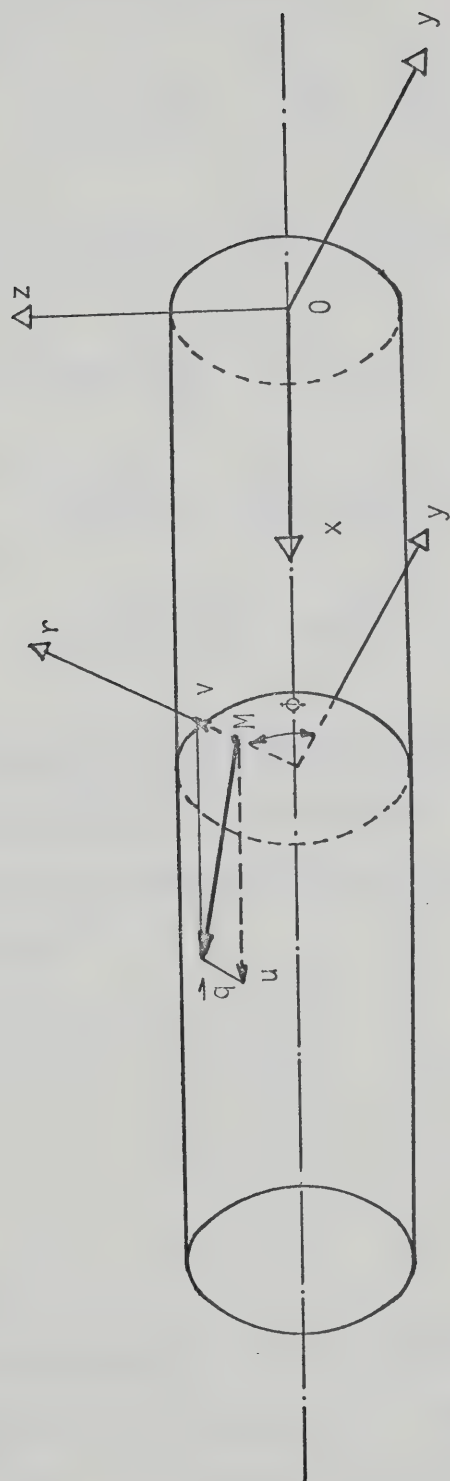


Figure 2.1 Coordinate System

2. Navier-Stokes equations

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] \quad (2.8)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} = - \frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right] \quad (2.9)$$

3. Energy Equation

$$\begin{aligned} c \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right] &= \frac{k}{\rho} \left[\frac{\partial^2 T}{\partial x^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right] \\ &+ \nu \left[\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r} \right)^2 + 2 \left(\left(\frac{\partial v}{\partial r} \right)^2 + \left(\frac{v}{r} \right)^2 + \left(\frac{\partial u}{\partial x} \right)^2 \right) \right] \end{aligned} \quad (2.10)$$

2.3.3 Deduction of the Parameters by Non-Dimensionalization Technique

The coordinates, the velocity components, the local temperature and pressure are non-dimensionalized by use of the following transformations:

$$\begin{aligned} x &= L \bar{x} & u &= U \bar{u} \\ r &= R \bar{r} & v &= V \bar{v} \\ T &= \Delta T \bar{T} & p &= \Delta P \bar{p} \end{aligned} \quad (2.11)$$

where the barred quantities are non-dimensional; U is the maximum longitudinal velocity component, V is the reference velocity in the radial direction, ΔP and ΔT are the maximum pressure drop and temperature difference in the entrance region respectively, R is the tube radius

and L the approximate length of the entrance region. The introduction of these quantities into the equations (2.7) through (2.10) allows the following set to be written:

$$\frac{\partial \bar{u}}{\partial x} + \left[\frac{VL}{UR} \right] \left(\frac{\partial \bar{v}}{\partial r} + \frac{\bar{v}}{r} \right) = 0 \quad (2.12)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{u}}{\partial x} + \left[\frac{VL}{UR} \right] \bar{v} \frac{\partial \bar{u}}{\partial r} = & - \left[\frac{\Delta p}{\rho U^2} \right] \frac{\partial \bar{p}}{\partial x} \\ & + \left[\frac{v}{UR} \right] \left[\frac{L}{R} \right] \left\{ \left[\frac{R}{L} \right]^2 \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}}{\partial r} \right\} \end{aligned} \quad (2.13)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{v}}{\partial x} + \left[\frac{VL}{UR} \right] \bar{v} \frac{\partial \bar{v}}{\partial r} = & - \left[\frac{UL}{VR} \right] \left[\frac{\Delta p}{\rho U^2} \right] \frac{\partial \bar{p}}{\partial r} \\ & + \left[\frac{v}{UR} \right] \left[\frac{L}{R} \right] \left\{ \left[\frac{R}{L} \right]^2 \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}}{\partial r} - \frac{\bar{v}}{r^2} \right\} \end{aligned} \quad (2.14)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{T}}{\partial x} + \left[\frac{VL}{UR} \right] \bar{v} \frac{\partial \bar{T}}{\partial r} = & \left[\frac{k}{\rho c U R} \right] \left[\frac{L}{R} \right] \left\{ \left[\frac{R}{L} \right]^2 \frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{T}}{\partial r} \right\} \\ & + \left[\frac{v}{c \Delta T} \right] \left[\frac{UL}{R^2} \right] \left\{ \left(\left[\frac{VR}{UL} \right] \frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial r} \right)^2 + 2 \left[\frac{v}{U} \right] \left(\left(\frac{\partial \bar{v}}{\partial r} \right)^2 + \frac{\bar{v}^2}{r^2} \right) \right. \\ & \left. + 2 \left[\frac{R}{L} \right]^2 \left(\frac{\partial \bar{u}}{\partial x} \right)^2 \right\} \end{aligned} \quad (2.15)$$

In order that both terms of the non-dimensionalized continuity equation (2.12) are of the same order of magnitude, set the factor $[VL/UR]$ equal to one; that is:

$$\frac{V}{U} = \frac{R}{L} \quad (2.16)$$

Substituting this relationship into the previous equations yields the following:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} = 0 \quad (2.17)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{r}} = & - \left[\frac{\Delta P}{\rho U^2} \right] \frac{\partial \bar{p}}{\partial \bar{x}} \\ & + \left[\frac{\nu}{UR} \right] \left[\frac{L}{R} \right] \left\{ \left[\frac{R}{L} \right]^2 \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} \right\} \end{aligned} \quad (2.18)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{r}} = & - \left[\frac{L}{R} \right]^2 \left[\frac{\Delta P}{\rho U^2} \right] \frac{\partial \bar{p}}{\partial \bar{r}} \\ & + \left[\frac{\nu}{UR} \right] \left[\frac{L}{R} \right] \left\{ \left[\frac{R}{L} \right]^2 \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\bar{v}}{\bar{r}} - \frac{\bar{v}}{\bar{r}^2} \right\} \end{aligned} \quad (2.19)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{r}} = & \left[\frac{k}{\rho c UR} \right] \left[\frac{L}{R} \right] \left\{ \left[\frac{R}{L} \right]^2 \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{T}}{\partial \bar{r}} \right\} \\ & + \left[\frac{U^2}{c \Delta T} \right] \left[\frac{\nu}{UR} \right] \left[\frac{L}{R} \right] \left\{ \left(\left[\frac{R}{L} \right]^2 \frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{r}} \right)^2 \right. \\ & \left. + 2 \left[\frac{R}{L} \right]^2 \left[\left(\frac{\partial \bar{v}}{\partial \bar{r}} \right)^2 + \left(\frac{\bar{v}}{\bar{r}} \right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 \right] \right\} \end{aligned} \quad (2.20)$$

From equations (2.18), (2.19) and (2.20) the following set of non-dimensional parameters can be deduced:

$$1. \text{ Length Ratio: } Le = \frac{L}{R}$$

$$2. \text{ Reynolds Number: } Re = \frac{UR}{\nu}$$

$$3. \text{ Euler Number: } Eu = \frac{\Delta P}{\rho U^2} \quad (2.21)$$

$$4. \text{ Peclet Number: } Pe = \frac{\rho c U R}{k}$$

$$5. \text{ Eckert Number: } E = \frac{U^2}{c \Delta T}$$

For the purpose of this thesis, which is not concerned with the pressure or temperature distribution throughout the bifurcation, the flow at the bifurcation inlet is fully described by only two of these dimensionless parameters, that is:

$$1. \text{ the Length Ratio: } Le = \frac{L}{R}$$

$$2. \text{ the Reynolds Number: } Re = \frac{UR}{\nu}$$

The Reynolds number characterizing the flow through a bifurcation is calculated at the entrance to the bifurcation. If the entry length upstream of the bifurcation is sufficient, the velocity profile at the bifurcation inlet is parabolic. This allows us to express the Reynolds number in terms of the average longitudinal velocity component u' . The average velocity is equal to half of the maximum velocity:

$$u' = \frac{U}{2} \quad (2.22)$$

and the Reynolds number becomes

$$Re = \frac{u'D}{\nu} \quad (2.23)$$

where D is the tube diameter.

CHAPTER III

APPARATUS AND INSTRUMENTATION

3.1 Requirements

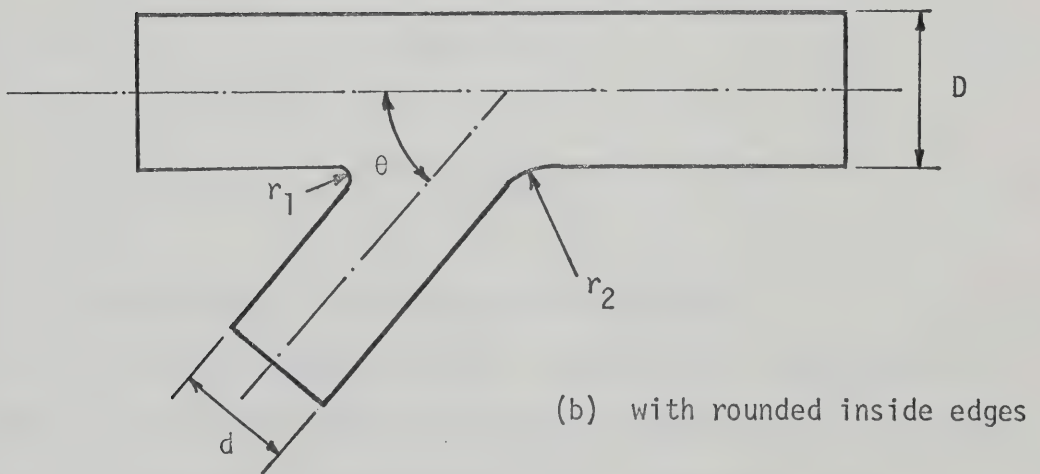
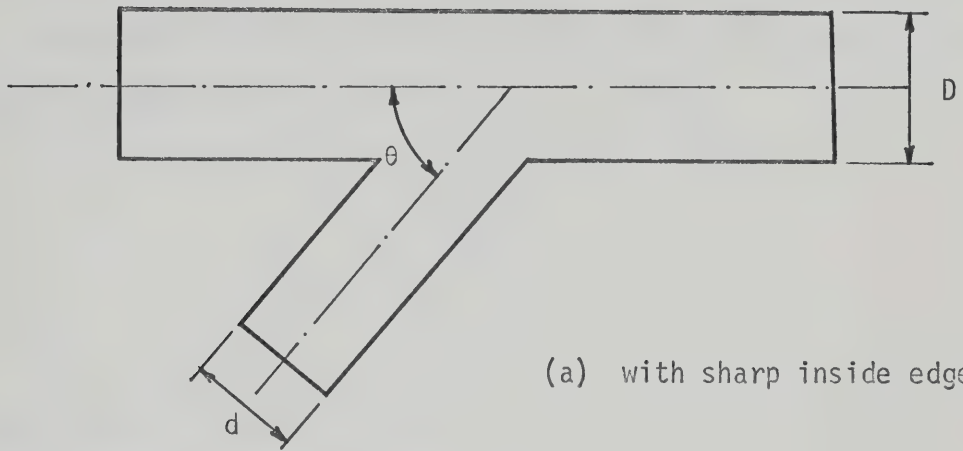
The purpose of the experimental model is to enable the study of the flow characteristics in arterial junctions. As explained in Chapter I, the present thesis confines itself to the steady case of blood flowing through a side-branch bifurcation. In order to determine the range of study, it is necessary to summarize the blood characteristics related to the present work. The following data has been collected by C.M. Rodkiewicz and D.H. Howell¹⁶:

1. Blood density varies from 1.048 to 1.066 g/cm³.
2. Blood relative viscosity with respect to distilled water varies from 3.5 to 5.4.
3. Blood average velocity through a 2.1 cm diameter aorta is 32 cm/s.
4. Human aorta diameter varies from 1.5 to 2.1 cm.

Substitution of this data into equation (2.23) yields the following entrance Reynolds number range:

$$1300 < Re < 2040 \quad (3.1)$$

The bifurcation geometry, as illustrated in Figure 3.1,



$$r_1 = \frac{3}{16} \text{ " } ; r_2 = \frac{3}{8} \text{ " }$$

Figure 3.1 Experimental Bifurcations

leads to the introduction of two further dimensionless geometrical parameters: α related to the angle of branching, θ , and β as the ratio of the side-branch internal diameter, d , to the mainline-branch internal diameter, D ; that is:

$$\alpha = \frac{\theta}{90^\circ} \quad (3.2)$$

$$\beta = \frac{d}{D} \quad (3.3)$$

The range of study for α and β are:

$$\frac{1}{3} \leq \alpha \leq 1 \quad (3.4)$$

$$0.40 \leq \beta \leq 1 \quad (3.5)$$

The main objectives of the experiments are:

1. To study the dependence of γ , ratio of the side-branch to the mainline-branch flow rates, on the three dimensionless numbers Re , α and β and to establish when and why γ is greater than one.
2. To establish, for each value of the couple (α, β) , the Reynolds number at which the boundary layer starts to separate from the wall and then to observe the variation in position of the separation point when the Reynolds number varies.
3. To determine the influence of the smoothness of the bifurcation edges on the flow.

4. To draw some conclusions regarding future research on the blood flow.

Finally, in order to have a better understanding of the flow evolutions, the experimental range for the Reynolds number has been extended to:

$$500 < Re < 5000 \quad (3.6)$$

3.2 Description of the Apparatus

3.2.1 Choice of the Experimental Fluid

Taking into consideration the simplifications of the physiological model, already used as basis of the mathematical approach, the flow of blood through a given bifurcation, for a given Reynolds number at its entrance, should be similar to the flow of any incompressible Newtonian fluid through a similar bifurcation, for the same entrance Reynolds number. For this reason, as well as for ease of utilization, water has been chosen as the experimental fluid.

In fact, in order to decrease the surface tension of the water with respect to the acrylic building material of the bifurcations, a wetting agent* and an antifoam** have been added to the water. Such a solution has a viscosity slightly different than water. The

*Photo-Flo 200 Kodak, Edmonton Photo Supply, Edmonton, Alberta.

**Anti-Foam Kodak, Edmonton Photo Supply, Edmonton, Alberta.

variation of its kinematic viscosity with the temperature has been measured with a Cannon-Fenske viscometer and reported in Appendix A.

3.2.2 Overall Description of the Apparatus

Figures 3.2 and 3.3 are the layout and a photograph of the apparatus, respectively. The apparatus can be divided into three different sections:

1. The flow generation unit
2. The flow measurement unit
3. The test section

These three sections are supported by a dexion frame and connected together by copper pipes or flexible tubings of 1.25 inch internal diameter (this diameter was chosen for ease of observation).

3.2.3 Flow Generation Unit (Figure 3.4)

The required steady flow is obtained by using a centrifugal pump* in series with a damping tank and a filter**. The damping tank is made of stainless steel and has an internal diameter of 10 inches and a height of 30 inches. Five sixths of this tank is filled with water, so that the remaining air cushion absorbs the possible fluctuations of the pump output. The outlet of the tank is located 20 inches above the bottom and is connected to the filter.

The total flow rate and consequently the entrance Reynolds

*Motor-Mount Centrifugal Pump Deming, Unit No. 137, Electrical Industries Ltd., Edmonton, Alberta.

**Honeycomb Filter Tube No. E15R10A, B. Guthrie Engineering Company Ltd., Edmonton, Alberta.

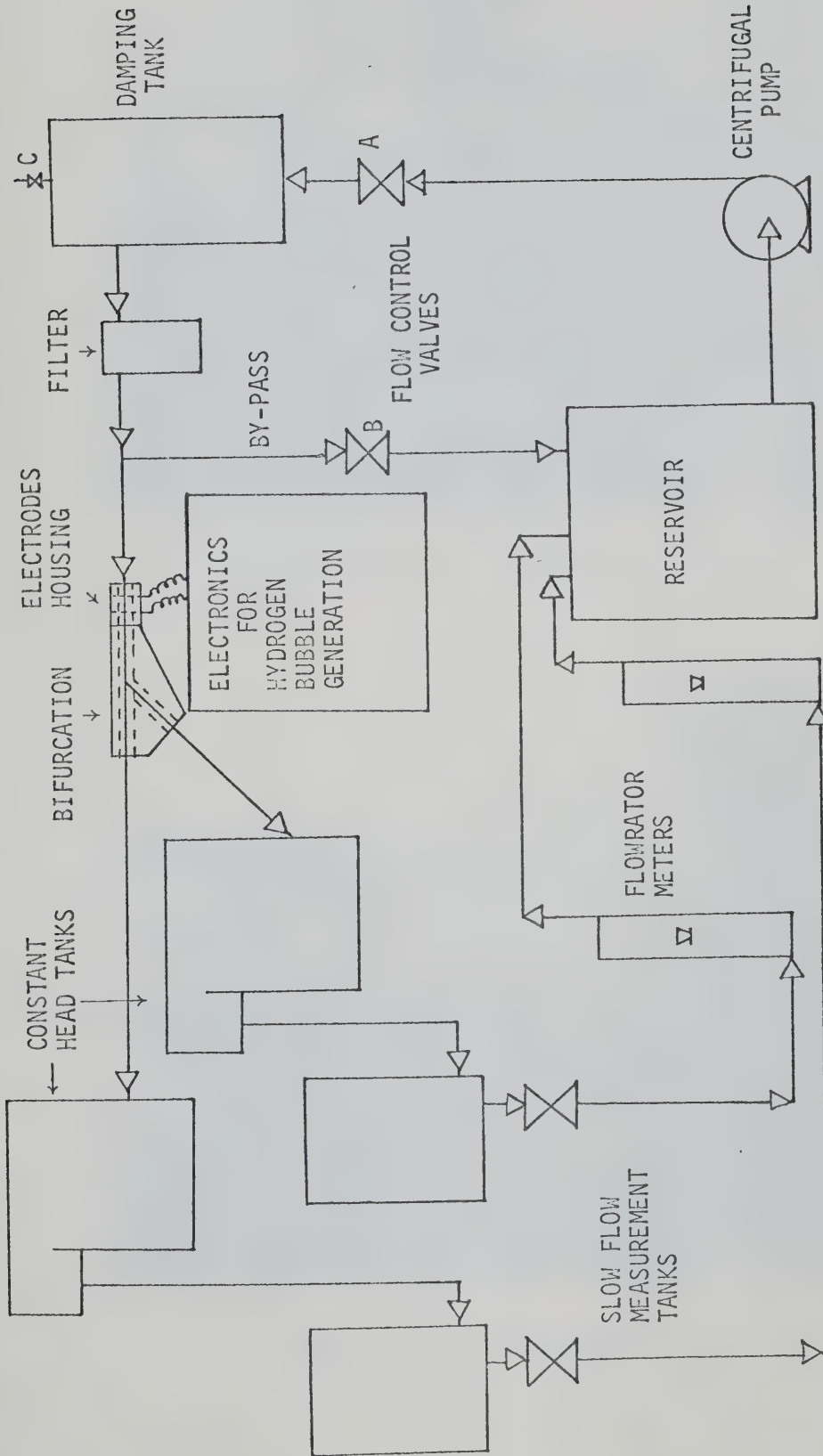


Figure 3.2 Apparatus Layout

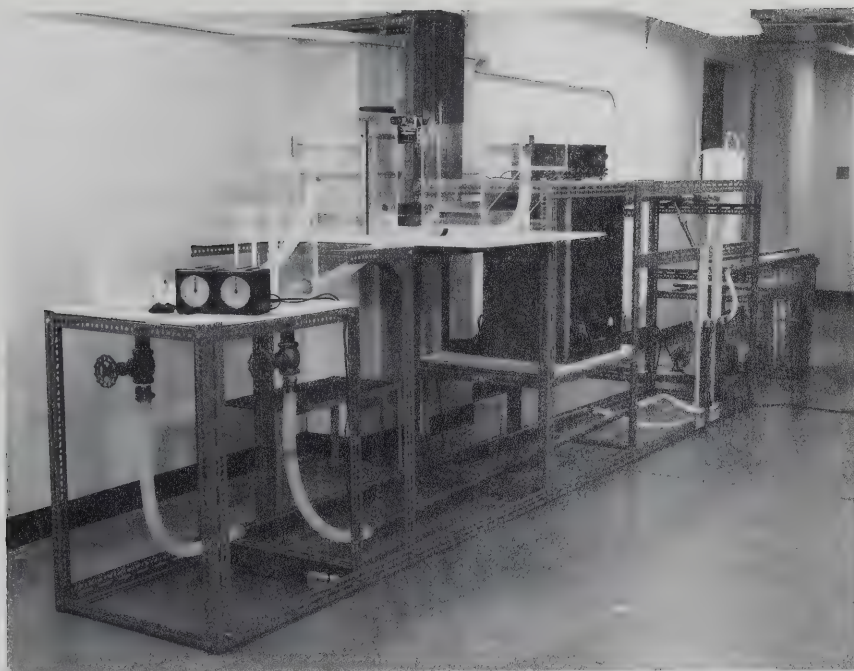


Figure 3.3 Photograph of the Apparatus

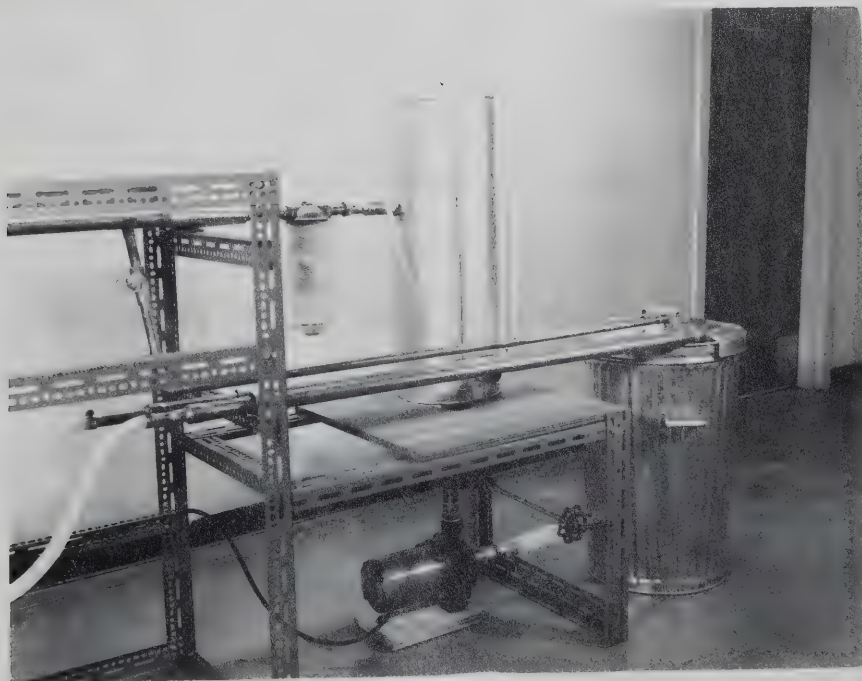


Figure 3.4 Flow Generation Unit

number is controlled by a 1-1/2" gate valve (A) at the outlet of the pump and by a 3/4" seat valve (B) located on the by-pass (valves A and B are shown on Figure 3.2). This flow generation unit is capable of delivering a maximum of 40 U.S. gallons per minute.

3.2.4 Flow Rate Measurement Unit (Figure 3.5)

As shown on the apparatus layout (Figure 3.2) and on the photographs (Figure 3.5), this unit has been divided into two different sections:

1. The slow flow measurement tanks.
2. The flowrator meters.*

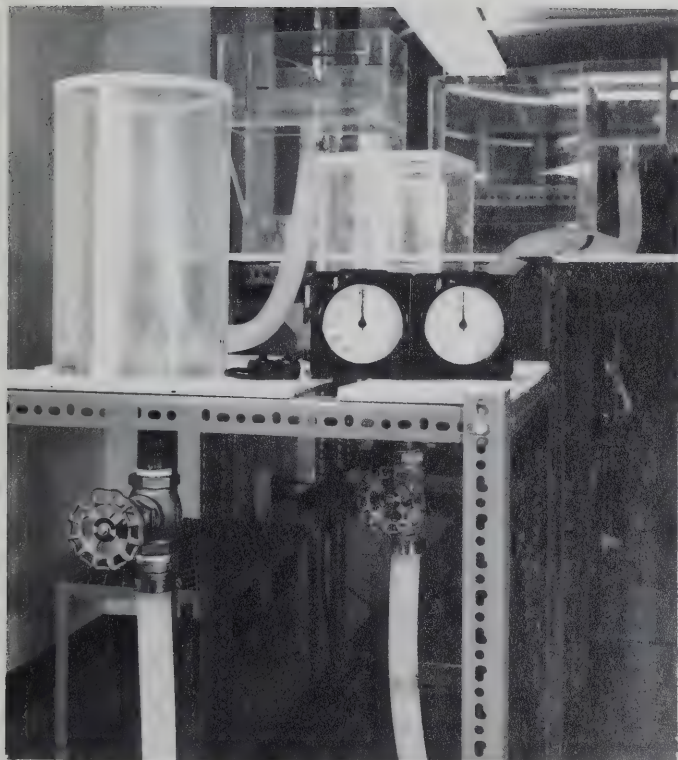
Low flow rates are determined, in each branch, by simultaneously closing the drains of the slow flow measurement tanks and measuring the elapsed time to collect a known quantity of water (varying from 1 to 5 liters). These tanks, made of clear plastic, were calibrated with extreme care.

When the flow is fast enough, the branch flow rates are measured by the flowrator meters, which have been specially calibrated to have an accuracy of one percent of the maximum flow. Each flow meter can measure flow rates up to 1.52 U.S. gallons per minute.

3.2.5 Test Section (Figure 3.6)

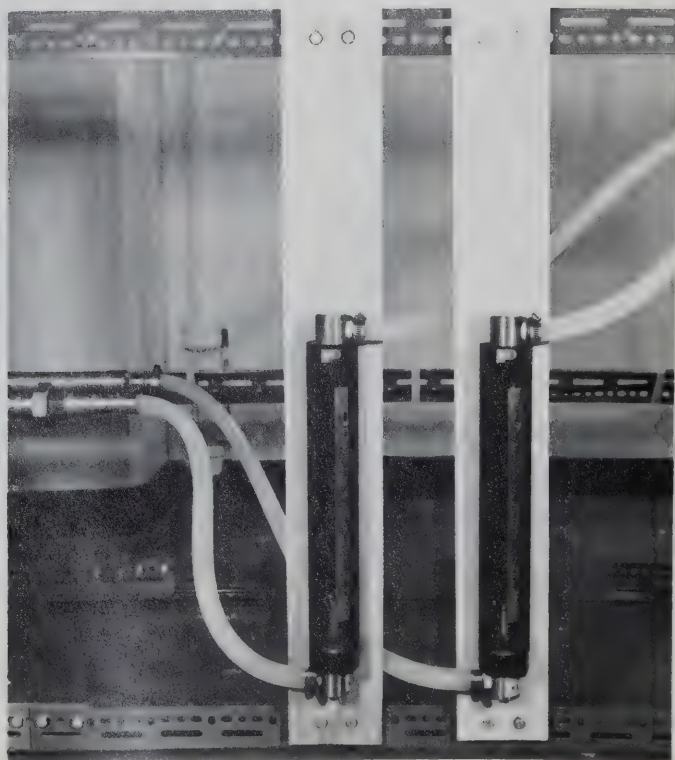
In order to enable observation and photography, the bifurcations, shown in Figure 3.8, were made of acrylic. Two kinds of bi-

*Flowrator Meter Fisher and Porter, Model 10A3565S Calibrated, Fisher & Porter (Canada) Ltd., Edmonton, Alberta.



(a) Slow Flow Measurement Tanks

Figure 3.5 Flow Rate Measurement Unit



(b) Flowrator Meters

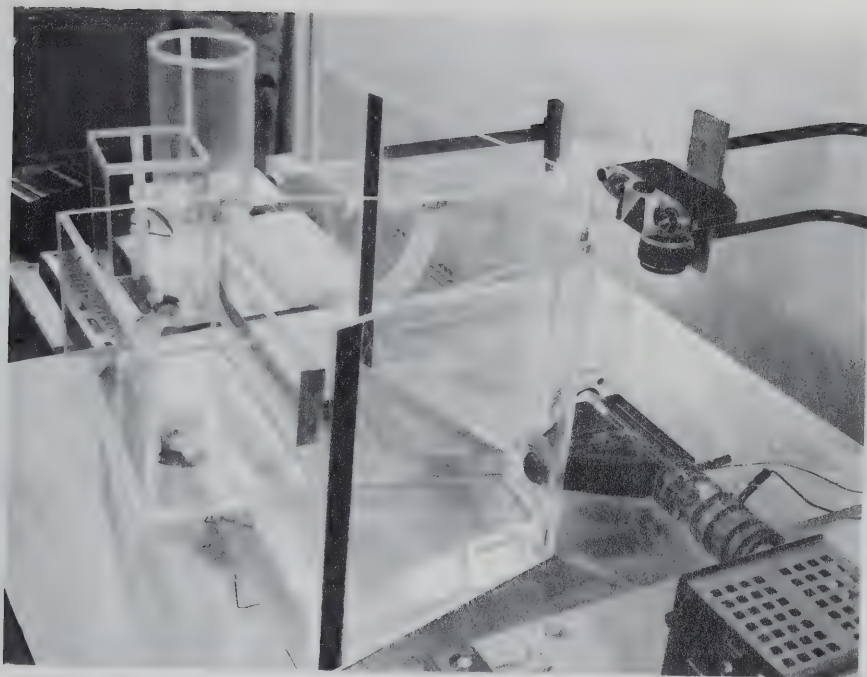


Figure 3.6 Test Section

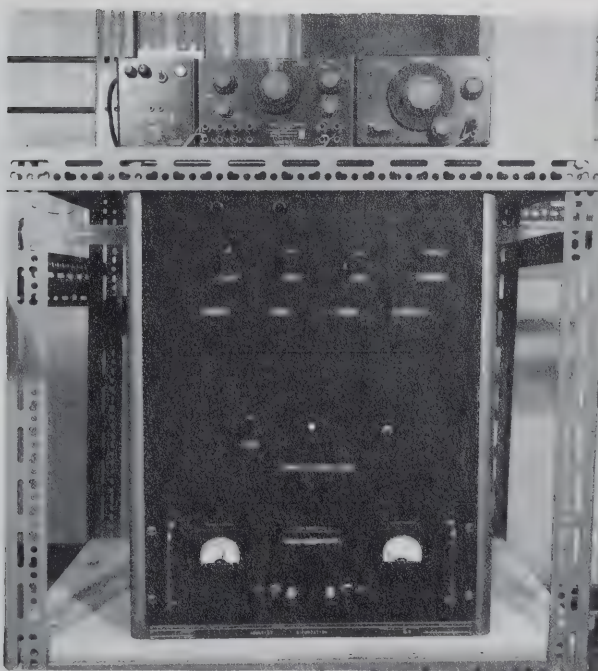


Figure 3.7 Generation Unit for Hydrogen Bubbles

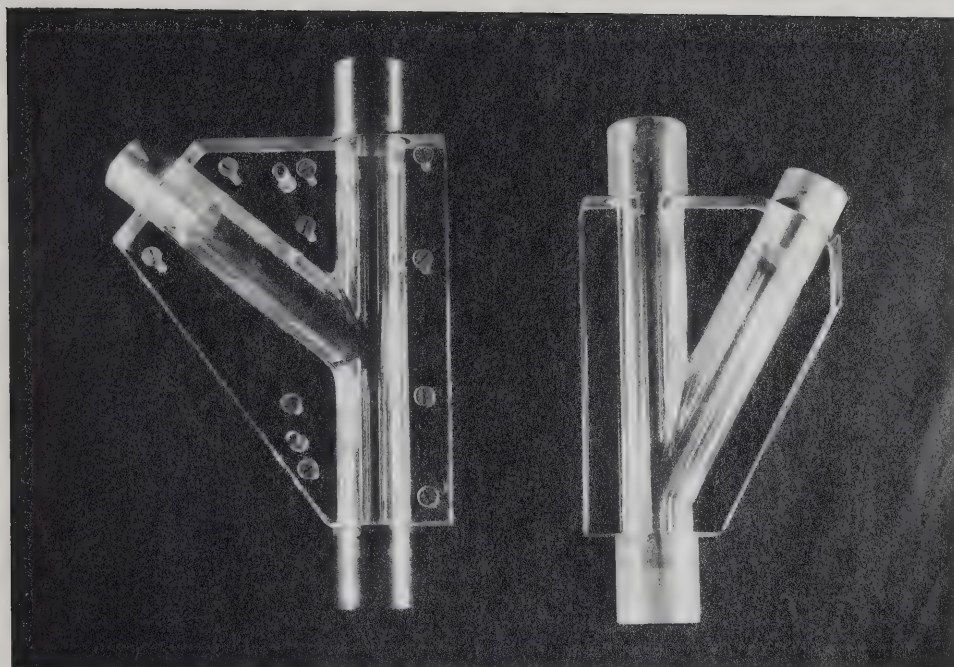


Figure 3.8 Two Typical Bifurcations

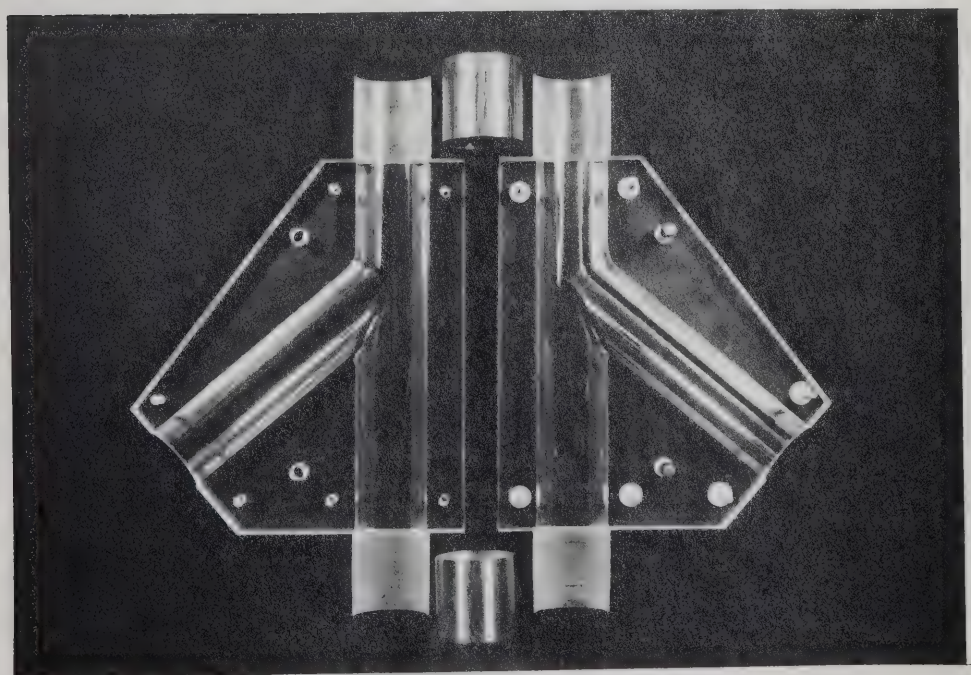


Figure 3.9 Disassembled Bifurcation with Rounded Edges

furcations have been built:

1. For the first kind, with sharp inside edges, the two branches were drilled in a 1-1/2" thick block of transparent plastic.

2. For the second kind, with rounded inside edges, two separate half-bifurcations were milled in 3/4" thick acrylic sheets (see Figure 3.9). The inside edges were then rounded by hand with curvature radius of 3/16" to 3/8" as illustrated in Figure 3.1. The two half bifurcations, positioned by two pins, were finally bolted together and sealed with chloroform.

The use of plastic blocks and sheets gives a flat upper outside surface to the bifurcations. This offsets serious optical distortions which would have been created by a curved air-plastic interface (see Figure 3.10). Table 3.1 shows the geometries and the corresponding dimensionless geometrical parameters of the tested bifurcations.

The inlet to the test section is a 1-1/4" internal diameter plastic pipe. Each bifurcation branch discharges into a constant head tank (see Figures 3.2 and 3.6). The two tanks have been designed to approximate the discharge of water from the bifurcation into two infinitely large reservoirs. They are identical, maintain the same water level at the two outlets of the bifurcation - 9" of water above the axis of each branch - and contain approximately 19 liters of water. The water overflows, through a 1 inch wide opening, from each reservoir to the corresponding measuring tank.

Table 3.1
Bifurcation Geometry

D(inches)	d(inches)	β	θ (degrees)	α
1.25	1.25	1	30	1/3
1.25	1.25	1	50	5/9
1.25	1.25	1	70	7/9
1.25	1.25	1	90	1
1.25	1.00	.80	30	1/3
1.25	1.00	.80	50	5/9
1.25	1.00	.80	70	7/9
1.25	1.00	.80	90	1
1.25	.75	.60	30	1/3
1.25	.75	.60	50	5/9
1.25	.75	.60	70	7/9
1.25	.75	.60	90	1
1.25	.50	.40	30	1/3
1.25	.50	.40	50	5/9
1.25	.50	.40	70	7/9
1.25	.50	.40	90	1

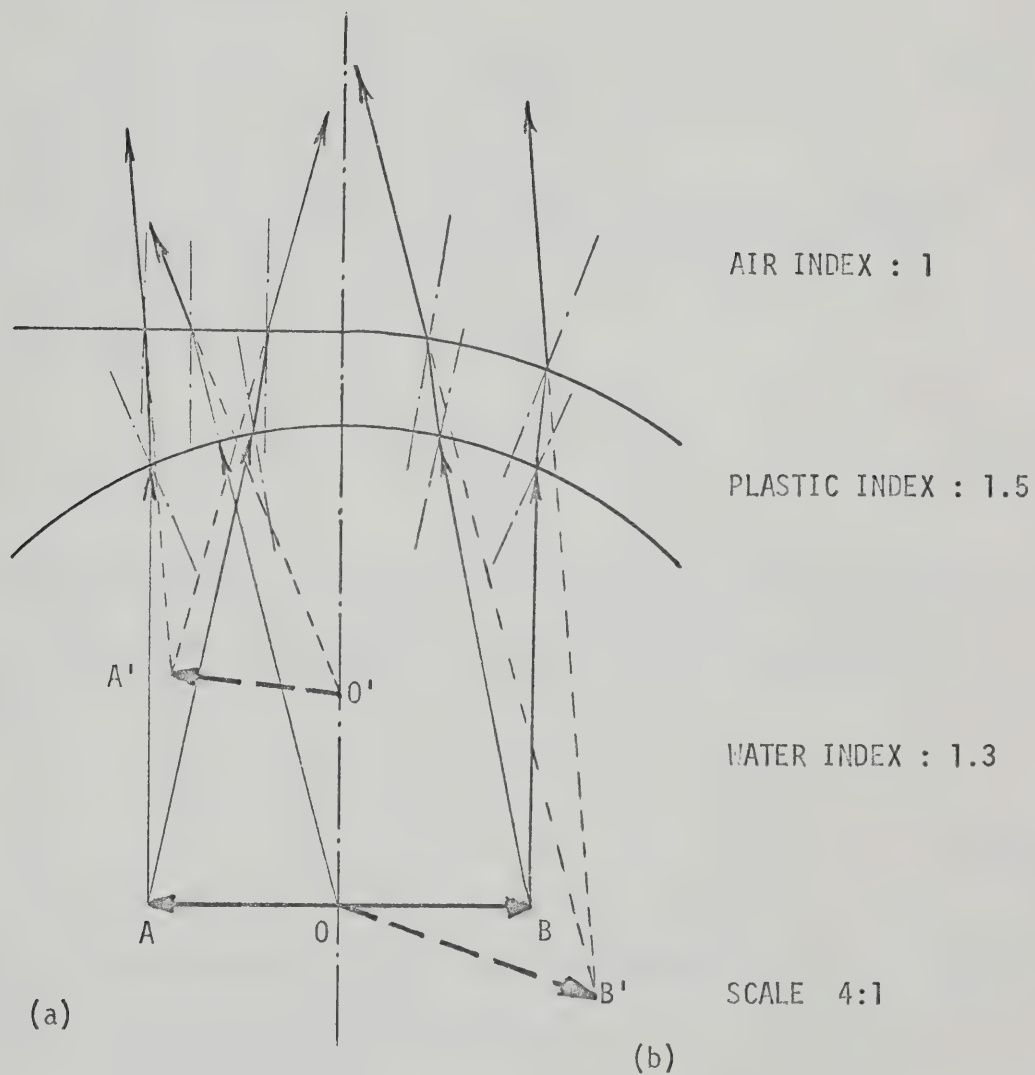


Figure 3.10 Optical Distortion Comparison Between Flat (a)
and Curved (b) Upper Outside Surface

3.2.6 Entrance Length

The upstream branch is 8'7" long, which gives a length ratio equal to:

$$Le = 164.8 \quad (3.7)$$

The coefficient $\sigma = 4 Le/Re$, as defined by H.L. Langhaar¹, corresponding to the critical Reynolds number for a straight pipe (see H. Schlichting¹⁷), is:

$$\sigma = 0.143 \quad (3.8)$$

According to reference 1 for this value of σ , the ratio of the maximum velocity to the mean velocity, at the bifurcation entrance, is given by:

$$\lambda = \frac{U}{u_1} = 1.93 \quad (3.9)$$

This means that the velocity profile, at the bifurcation inlet, approximates the parabolic profile within 3.5%.

One can notice that

1. This approximation shrinks down to 0.50% when the Reynolds number is 1650.

2. In order to approximate the parabolic profile within 0.50% for a Reynolds number of 2300, the entry length would have to be 16'4",

which was not practical because of space limitations and would be of doubtful benefit.

3.3 Flow Visualization

One of the best qualitative as well as quantitative methods of visualizing water flow is the "Hydrogen Bubble Technique".

3.3.1 Introductory Remarks on Electrolysis

A typical electrolysis circuit is shown in Figure 3.11.

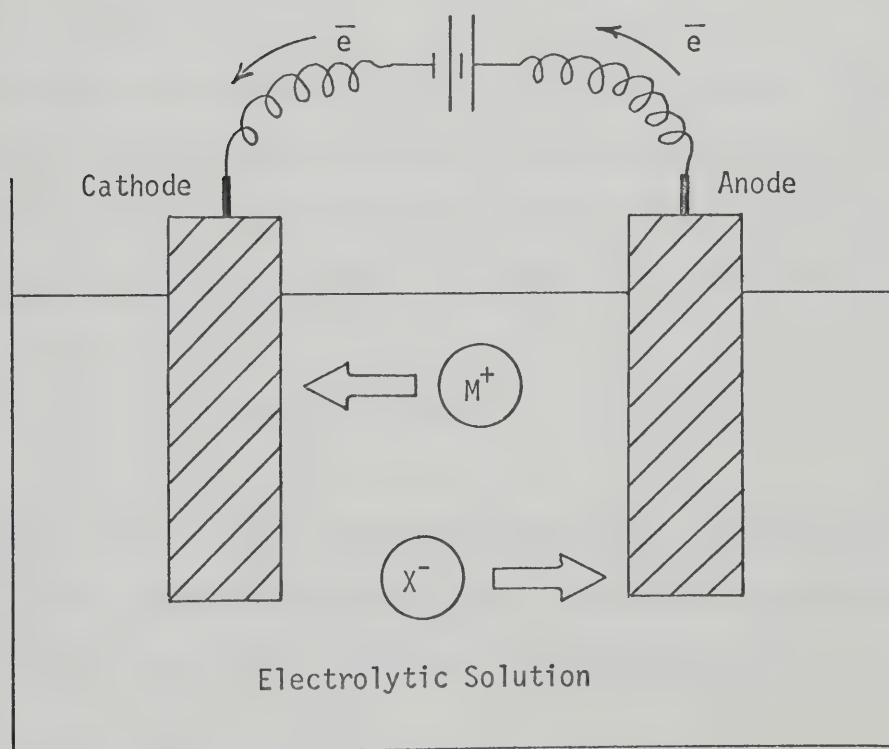
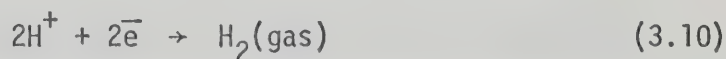


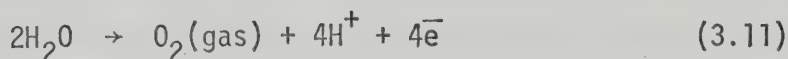
Figure 3.11 Electrolysis

When the battery or the DC power supply is operating, the electrons, drained away from the anode, are crowded onto the cathode. The electric circuit can only be closed if, simultaneously, a reduction and an oxidation occur respectively at the cathode and the anode.

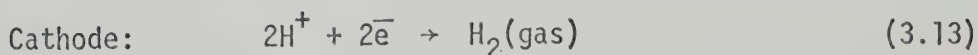
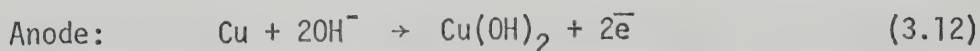
Using water as the electrolytic conductor, when a metal with a positive oxidation potential is chosen as a cathode material, the only possible reduction at this electrode will be:

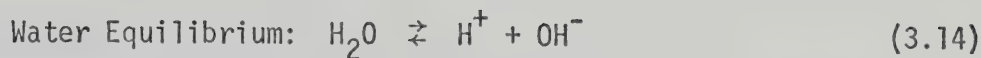


The formation of oxygen gas from water by the equation (3.11) has an oxidation potential equal to -1.23V. If the anode is made of a metal with an oxidation potential greater than -1.23V, no oxygen can appear at the anode, since the oxidation of the anode is more likely to occur.



Consequently the use of a tungsten cathode and a copper anode will yield to the following reactions, since the oxidation potentials of tungsten and copper are + 0.12V and -0.34V, respectively:





3.3.2 Hydrogen Bubble Technique

The hydrogen bubble technique utilizes the electrolysis of water to introduce hydrogen into the flow. These bubbles, formed at the fine-wire cathode (0.002" diameter), are swept off by the flow. This 0.002" diameter wire produces 0.001" diameter bubbles¹⁸, which have a maximum vertical velocity, due to buoyancy effect (which increases with the size of the bubbles) of 0.0011 fps. and reach a velocity of 98 percent of the fluid velocity within 1 ms, as explained by W. Davis and R.W. Fox¹⁹.

The wire has, due to its size, an insignificant influence on the flow itself. This constitutes one of the best advantages of this method, which also provides a great flexibility to yield qualitative and quantitative conclusions. Qualitative results can be obtained by continuously producing bubbles. This visualizes the pathlines, which are also streamlines and streaklines for a steady flow. If the bubbles are generated by electrical pulses, with a known frequency, one can simultaneously measure the velocities over a whole plane at a given instant.

The success of this visualization method is highly dependent on the correct illumination. A sharply collimated sheet of powerful light is necessary as well as dark backgrounds and suppression of other light sources.

3.3.3 Experimental Equipment

A special electrodes housing (Figure 3.12) has been built to hold both cathode (a 0.002" diameter tungsten wire) and anode (a copper ring). This housing has been designed so that:

1. It perfectly matches the internal diameters of both upstream pipe and bifurcation inlet.
2. It can rotate around its axis in order to generate sheets of bubbles in any of the diametrical planes at the inlet to the bifurcation.
3. It enables the change of wire.

The electrolysis DC current is brought to these electrodes from the generation unit (see Figures 3.7 and 3.13), which allows the operator to have complete and precise control of the frequency and the duration of the pulses producing the bubbles. The main part of this generation unit is the constant DC potential power supply*. Its 360V are applied between the two electrodes each time as long as the electronic switch (see diagram on Figure 3.14) is turned on by the pulse generator** driven by an oscillator[†]. The oscillator regulates the frequency of the pulses, while the pulse unit controls their duration. Finally a reversing switch has been installed so that the wire

*Lambda Regulated Power Supply, Model C 882M, Lambda Electronics Corp.

**Unit Pulse Generator, Type No. 1217C, General Radio Company.

[†]Oscillator 2C-2MC, Type No. 1310A, General Radio Company.

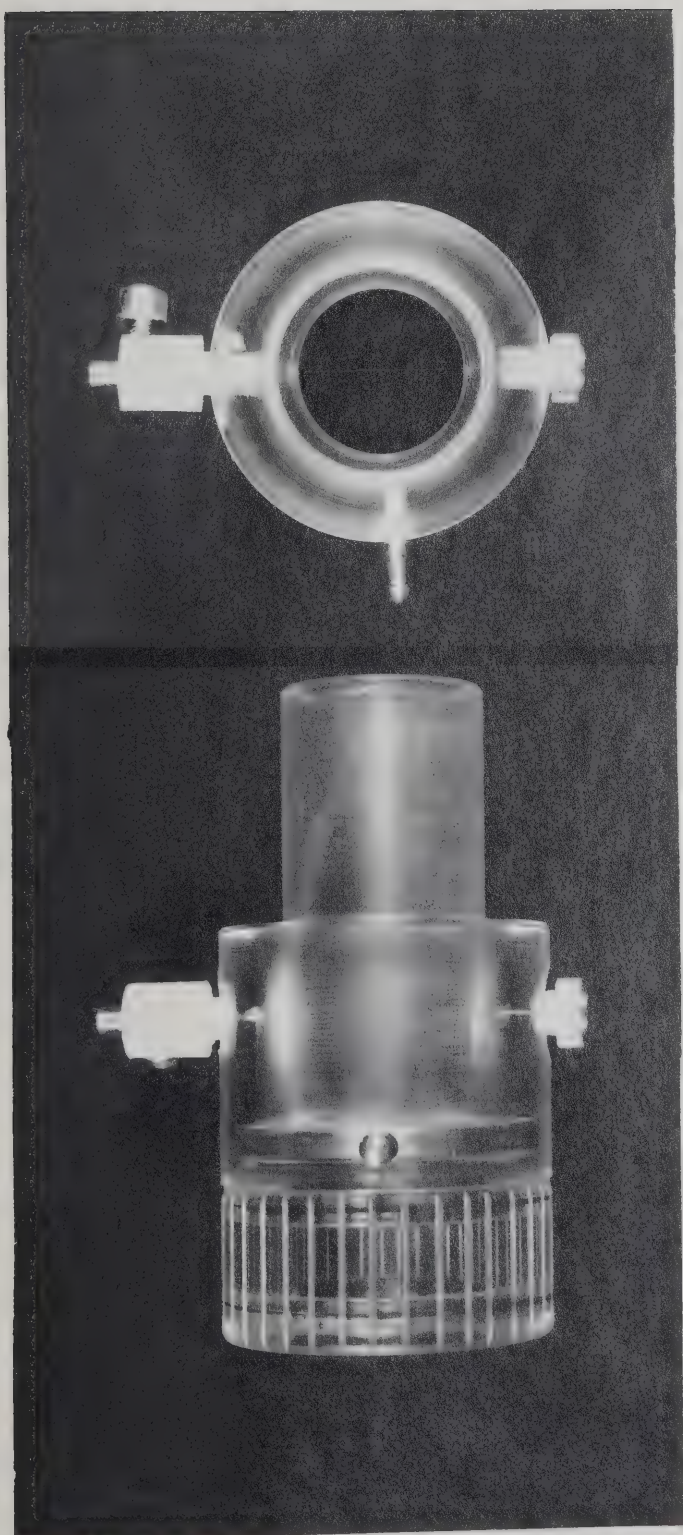


Figure 3.12 Electrodes Housing

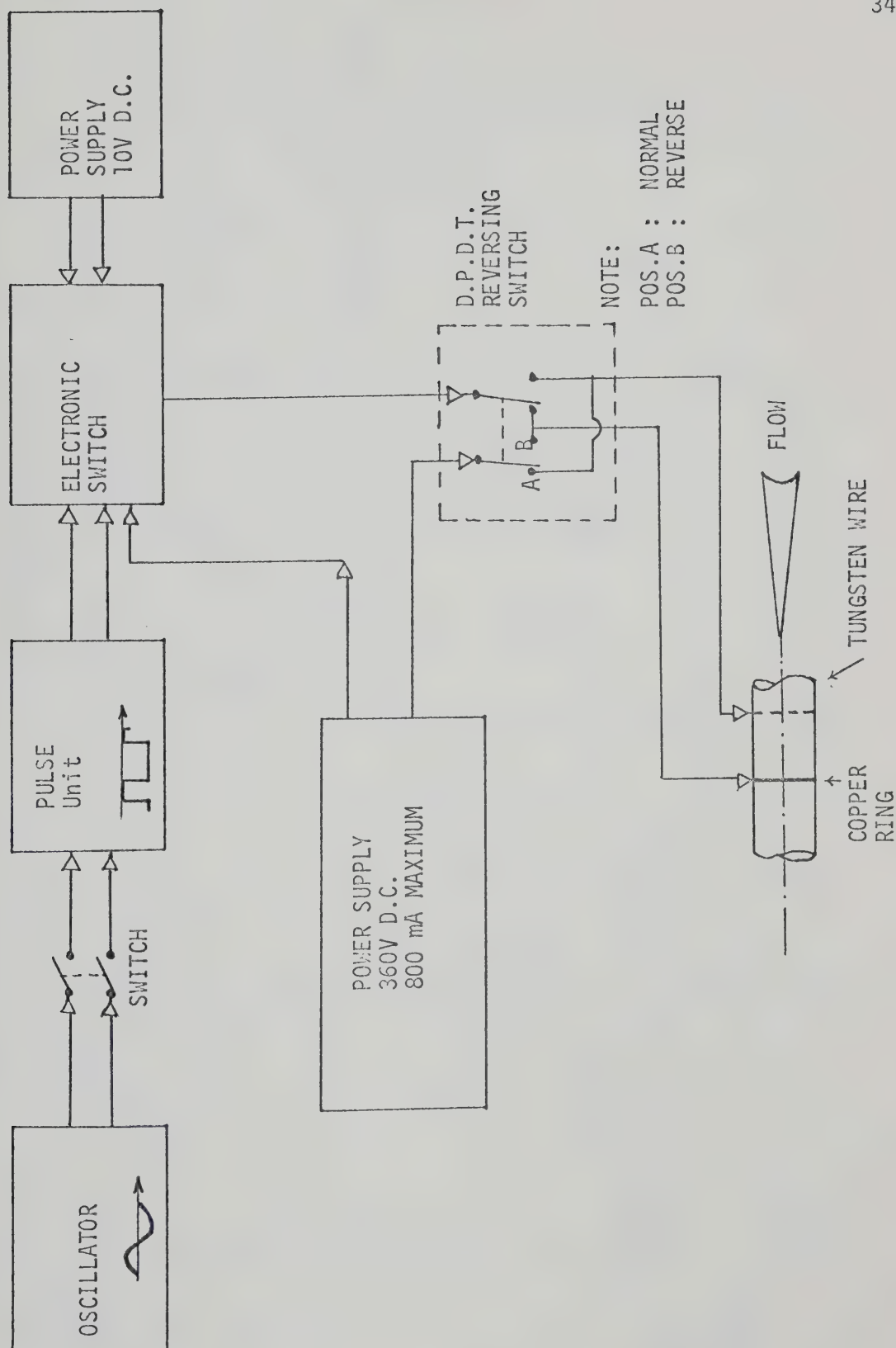


Figure 3.13 Electronics for Hydrogen Bubble Generation

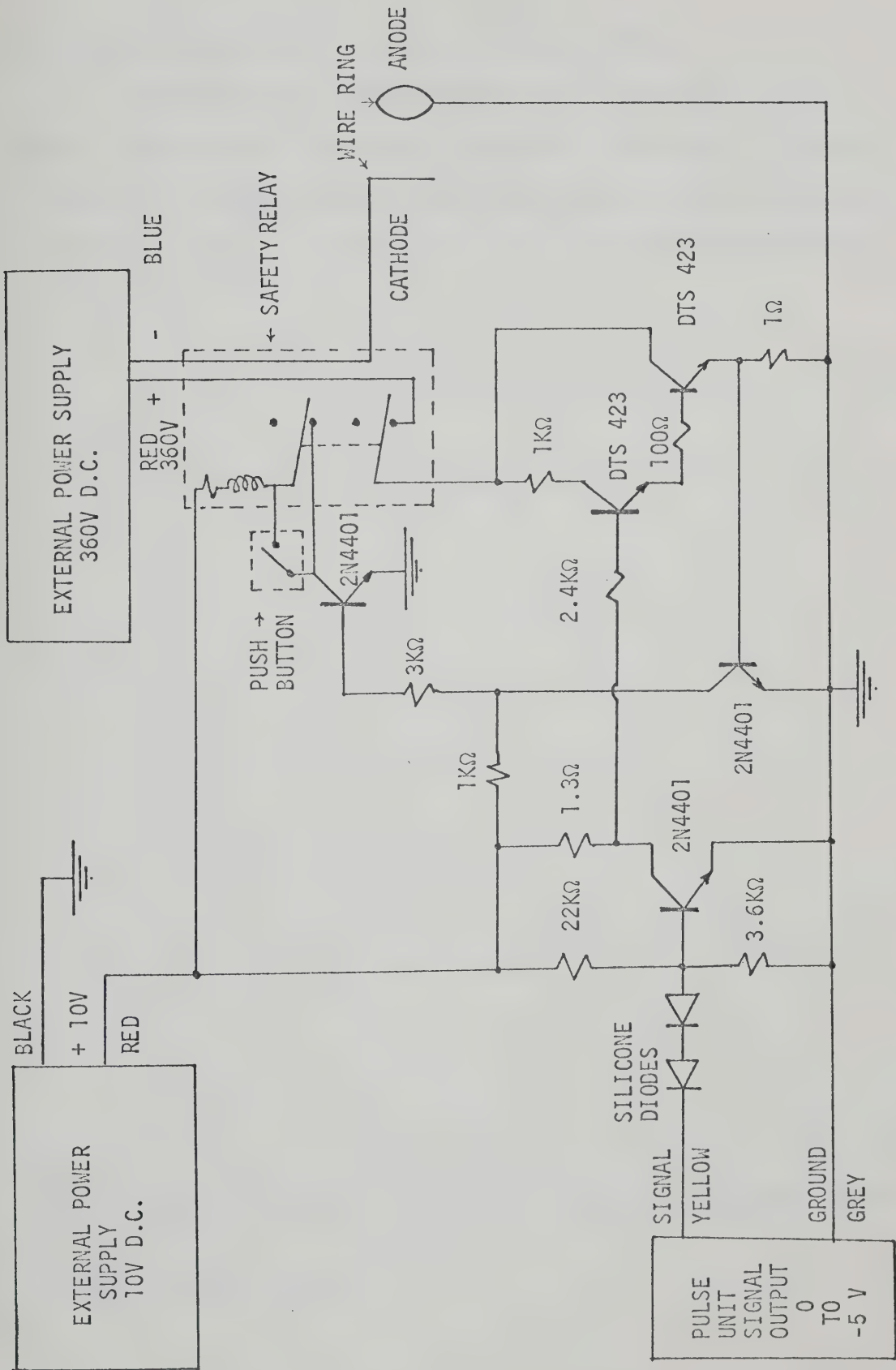


Figure 3.14 Electronic Switch - (Output Current Limited to 600mA)

can be cleaned by reversing the electrodes polarity.

Photographs of the streamlines are taken with a reflex camera* loaded with a high speed film** while the proper illumination is obtained by use of two slide projectors (total power 450W) giving a collimated sheet of light in the same plane as the bubbles sheet.

*Miranda Camera with Auto-Miranda Lens 1:1.9, $f = 50$ mm and Extension Tube.

**Kodak Tri-X Pan, ASA 400.

CHAPTER IV

EXPERIMENTS, RESULTS AND DISCUSSION

4.1 Experimental Procedure

4.1.1 Preliminary Procedure

This preliminary procedure, done after exchange of the filtering element is:

1. If not already in place, the appropriate bifurcation is installed.
2. The pump is started, while valves A and C are opened and valve B is closed (see Figure 3.2). The damping tank is then under atmospheric pressure.
3. When the damping tank is $5/6$ full of water, the valve C is closed.

4.1.2 Actual Experimental Procedure

The two first steps of the following procedure are to be disregarded, when using the preliminary procedure. This actual experimental procedure is:

1. The appropriate bifurcation is installed.
2. The pump is started, while valves B and C are closed and valve A is opened.
3. When the water almost overflows the constant head tanks valve A is closed and the pump is stopped.

4. After the system reaches equilibrium, the levels of the two constant head tanks are adjusted to be exactly the same. This constitutes the most important step, as far as coherence in the results is concerned.

5. The pump is then started and the flow is set as slow as possible by the use of the flow control valves A and B (see Figure 3.2).

6. The electronics for hydrogen bubble generation are turned on.

7. The flow rates in the two bifurcation branches are measured using the slow flow measurement tanks.

8. The temperature of the solution is measured in one of the two constant head tanks.

9. Observation of the flow through the bifurcation is made and photographs are eventually taken.

10. The total flow rate is increased.

11. Operations 7 through 10 are repeated until the flow is fast enough to be measured by the flow rator meters with sufficient precision.

12. Operations 8 through 11 are repeated until the flow rates are no longer within the range of study.

13. In order to drain the test section, valves A and B are closed and opened, respectively, while the pump is stopped.

The entrance Reynolds number is calculated from the values of the total flow rate and of the temperature of the solution by the use of either coefficient a or coefficient b, given in Appendix A.

4.2 Experimental Results

The experimental data are given in Appendix B.

4.2.1 Mass Flow Ratio, γ , as Function of Reynolds Number, Re

Figures 4.1 through 4.4 and 4.5 through 4.8 give the variation of the mass flow ratio, γ , with respect to the Reynolds number, Re , as a function of the diameter ratio, β , for the sharp and round edged bifurcations, respectively.

Figures 4.9 through 4.12 and 4.13 through 4.16 give the variation of γ with respect to Re as a function of α for the sharp and round edged bifurcations, respectively.

Figure 4.17 gives a comparison between the two kinds of bifurcations for $\alpha = 5/9$.

4.2.2 Mass Flow Ratio, γ , as Function of the Bifurcation Geometry

Figures 4.18 and 4.19 give the variation of γ with respect to β as a function of Re for the sharp and round edged bifurcations with $\beta = 5/9$, respectively.

Figures 4.20 and 4.21 give the variation of γ with respect to α as a function of Re for the sharp and round edged bifurcations with $\beta = 0.60$, respectively.

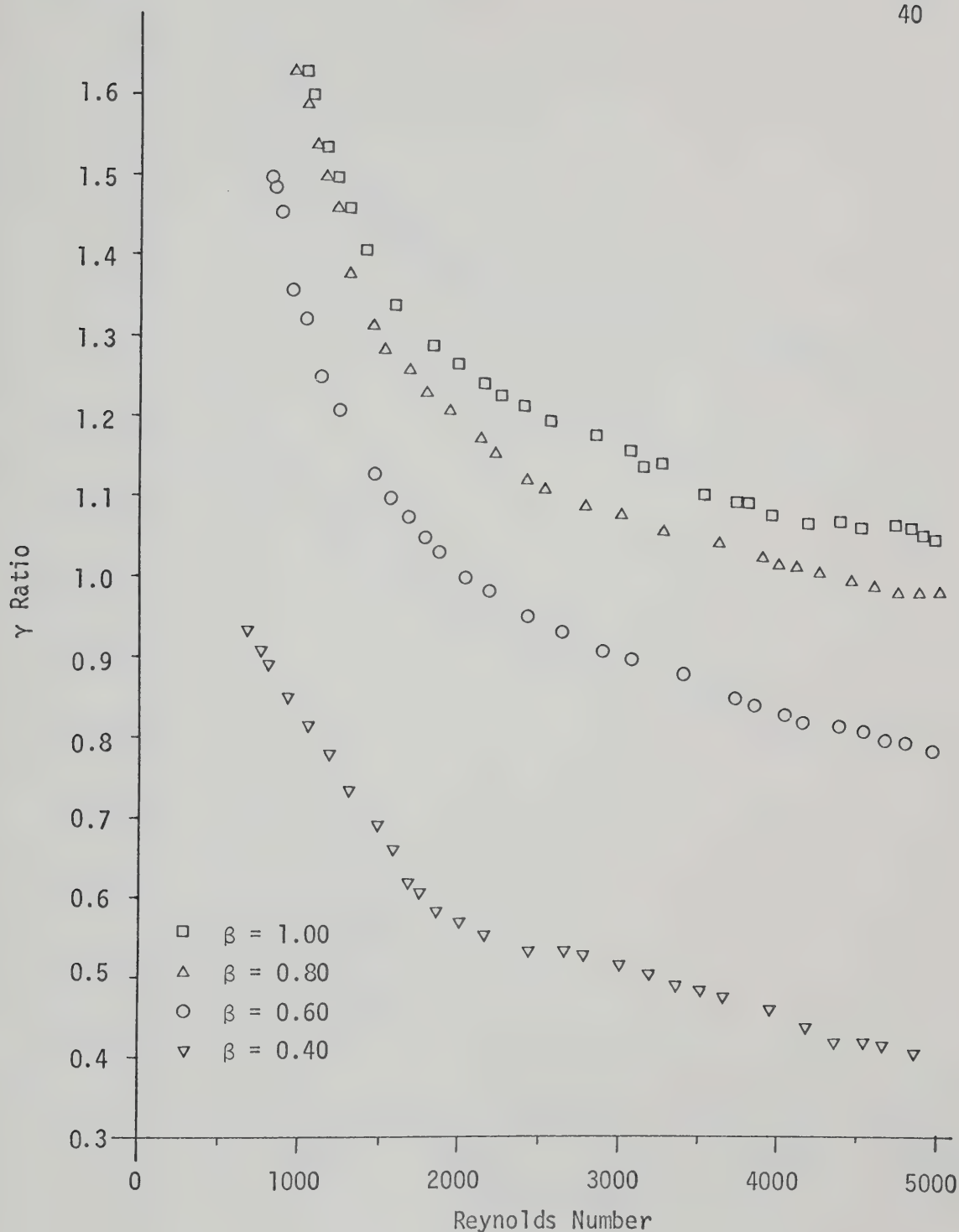


Figure 4.1 Sharp Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of β for $\alpha = 1/3$ ($\theta = 30^\circ$)

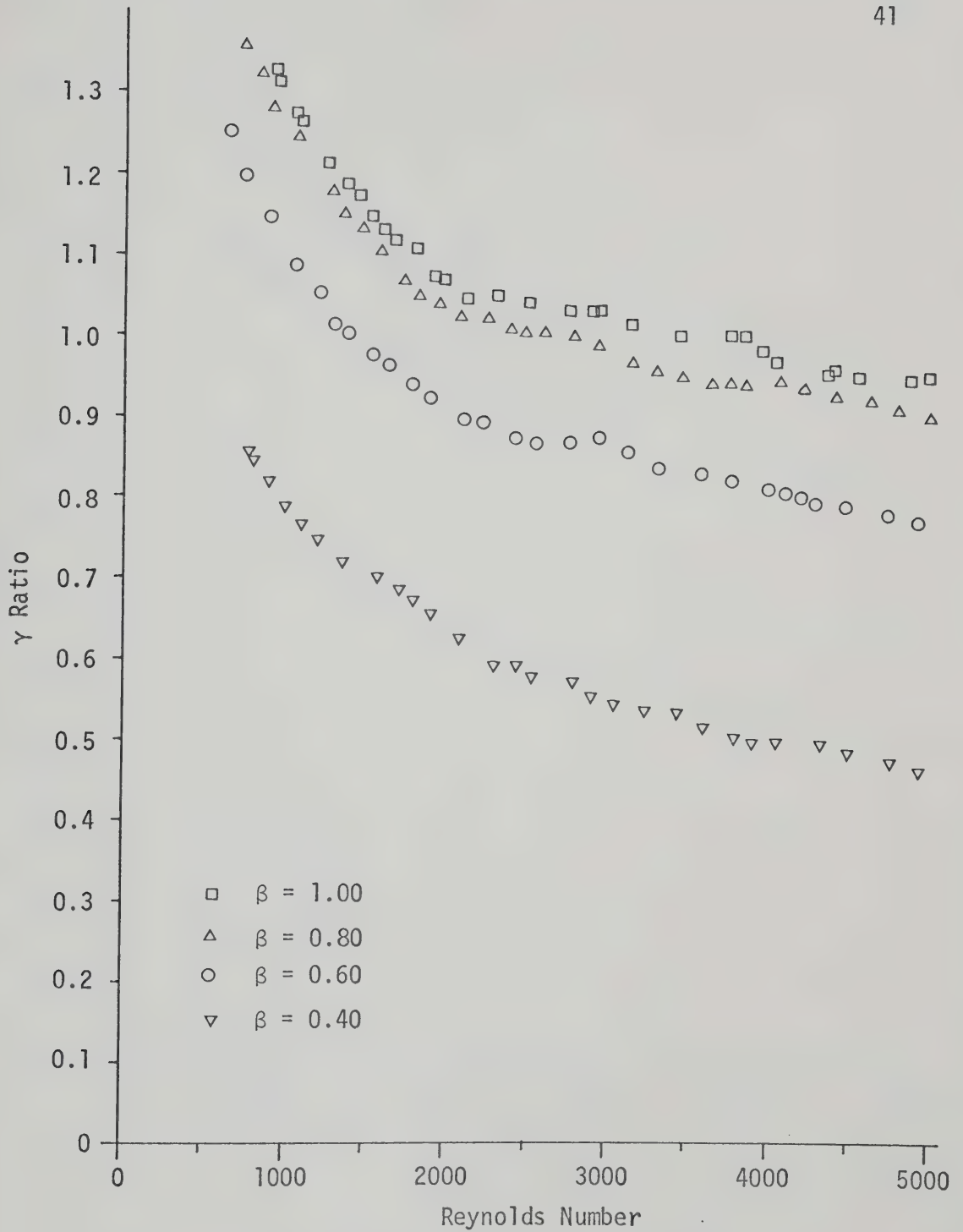


Figure 4.2 Sharp Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of β for $\alpha = 5/9$ ($\theta = 50^\circ$)

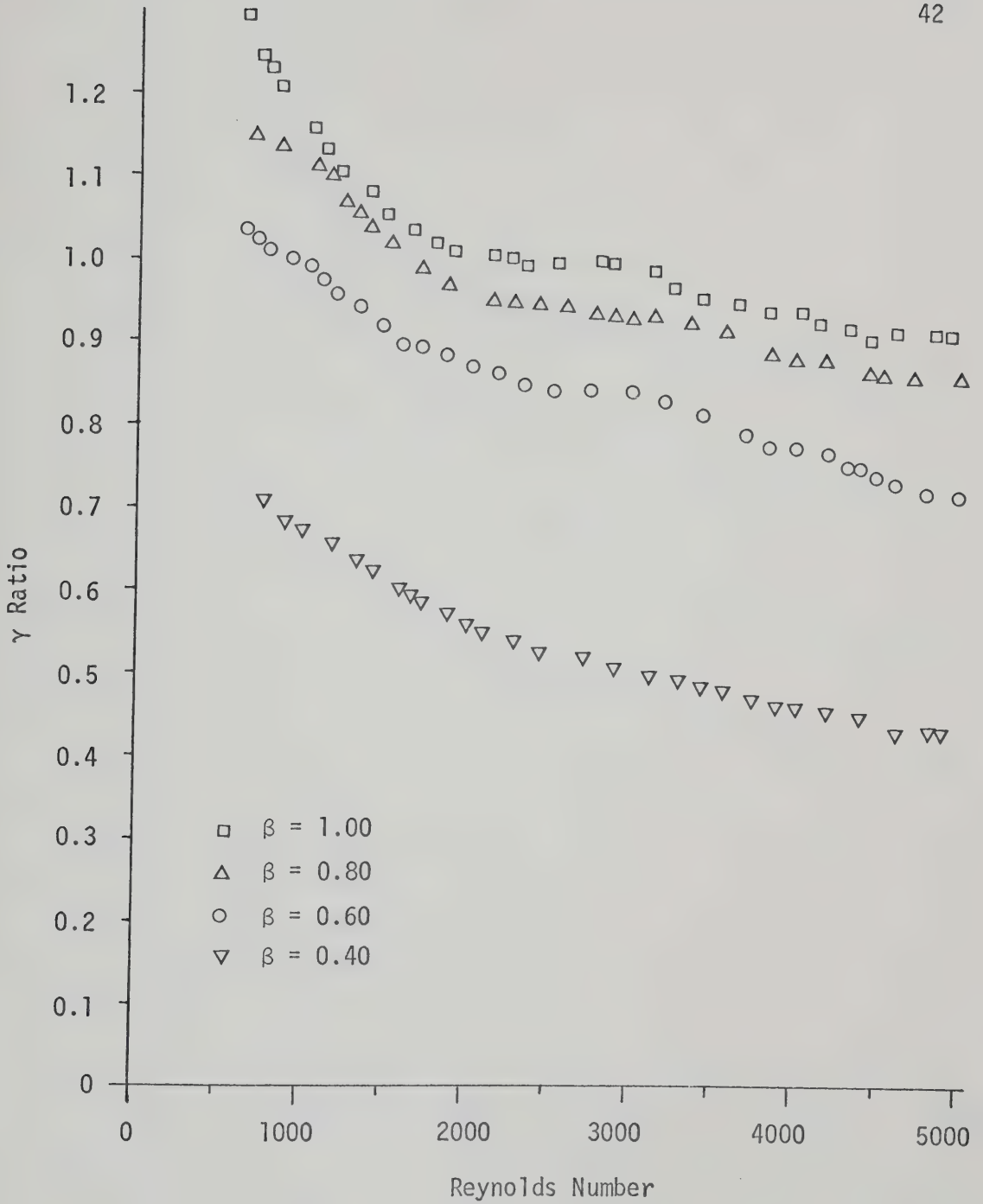


Figure 4.3 Sharp Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of β for $\alpha = 7/9$ ($\theta = 70^\circ$)

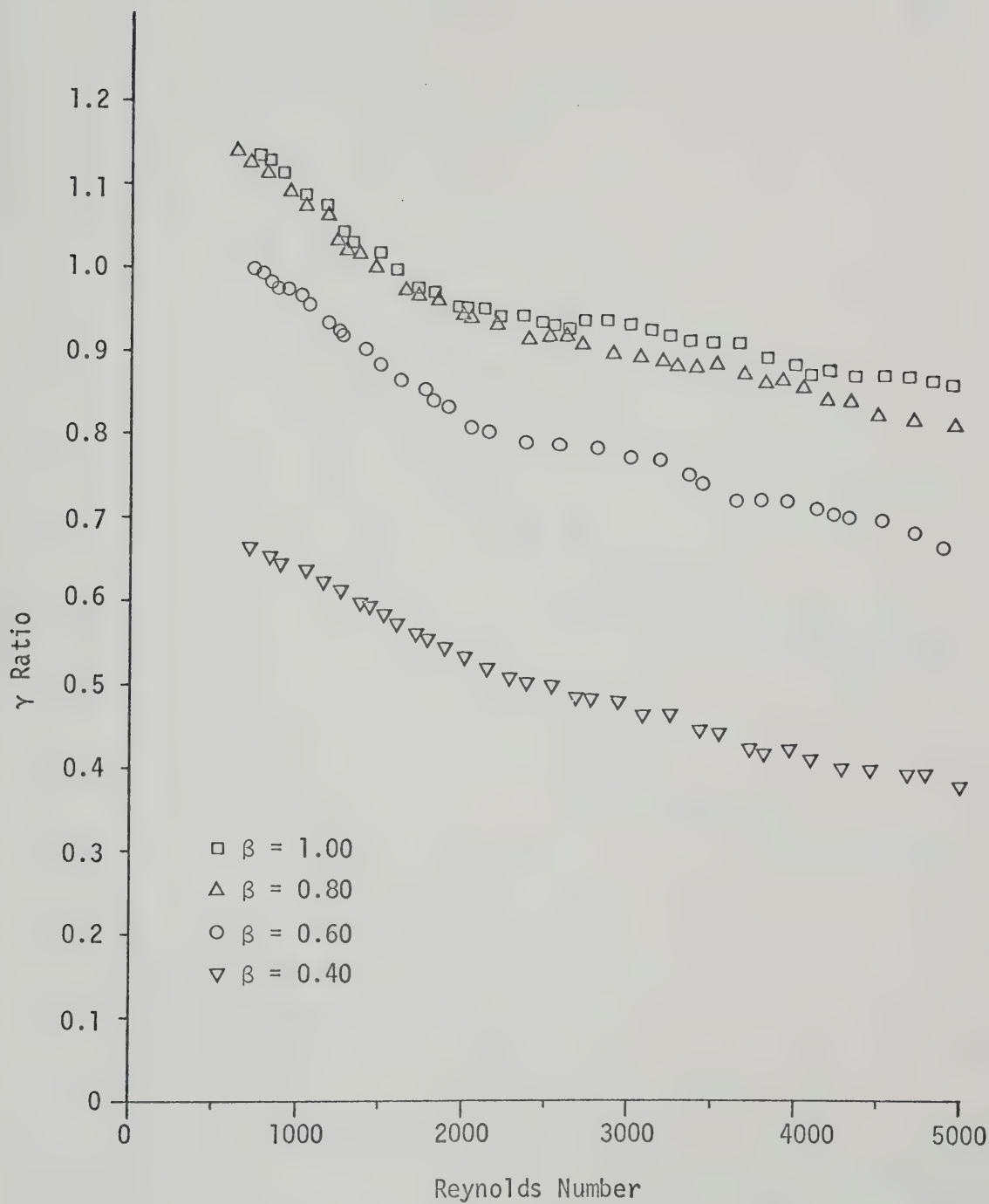


Figure 4.4 Sharp Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of β for $\alpha = 1$ ($\theta = 90^\circ$)

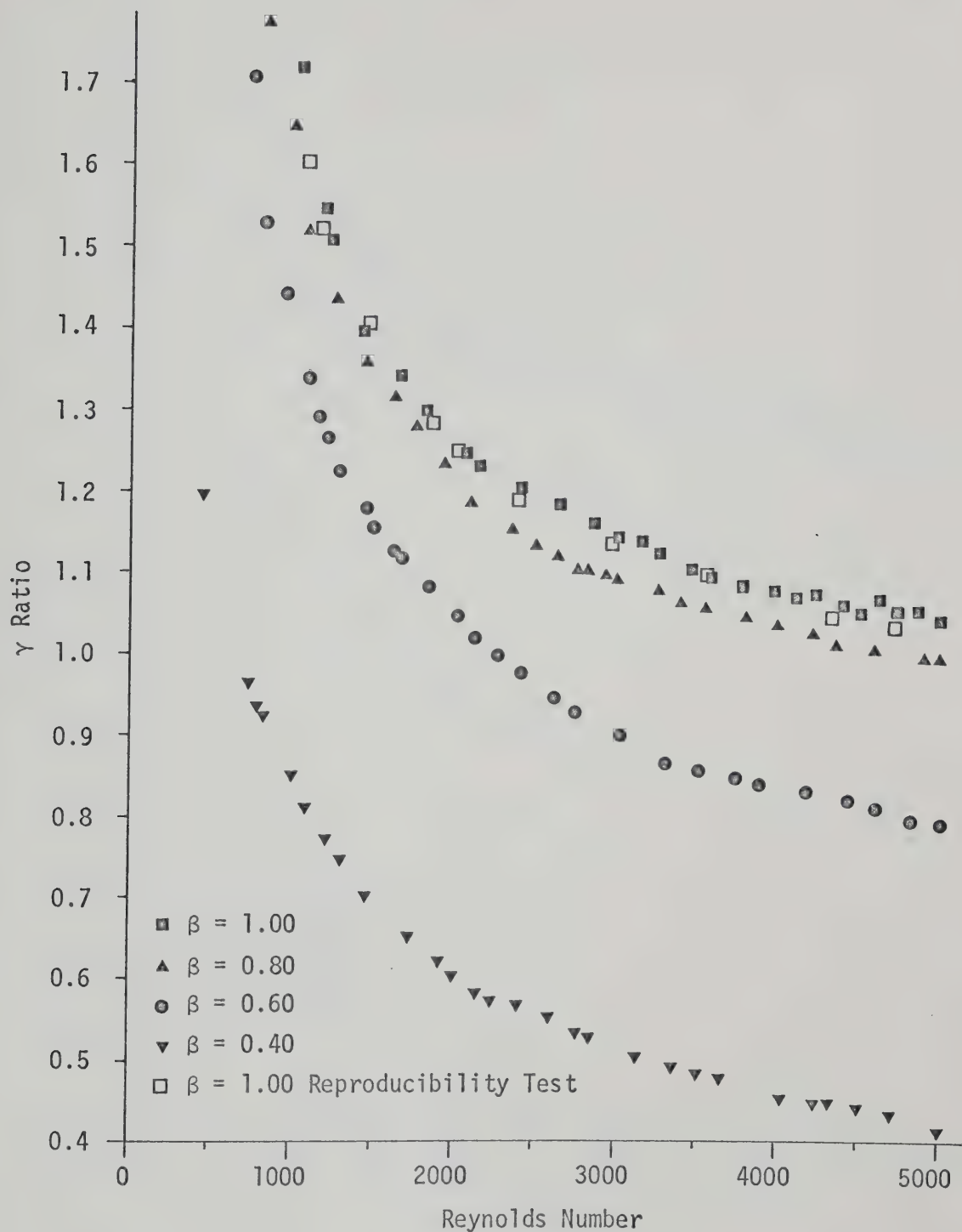


Figure 4.5 Round Edged Bifurcation
 Mass Flow Ratio γ versus Reynolds Number Re
 as a Function of β for $\alpha = 1/3$ ($\theta = 30^\circ$)

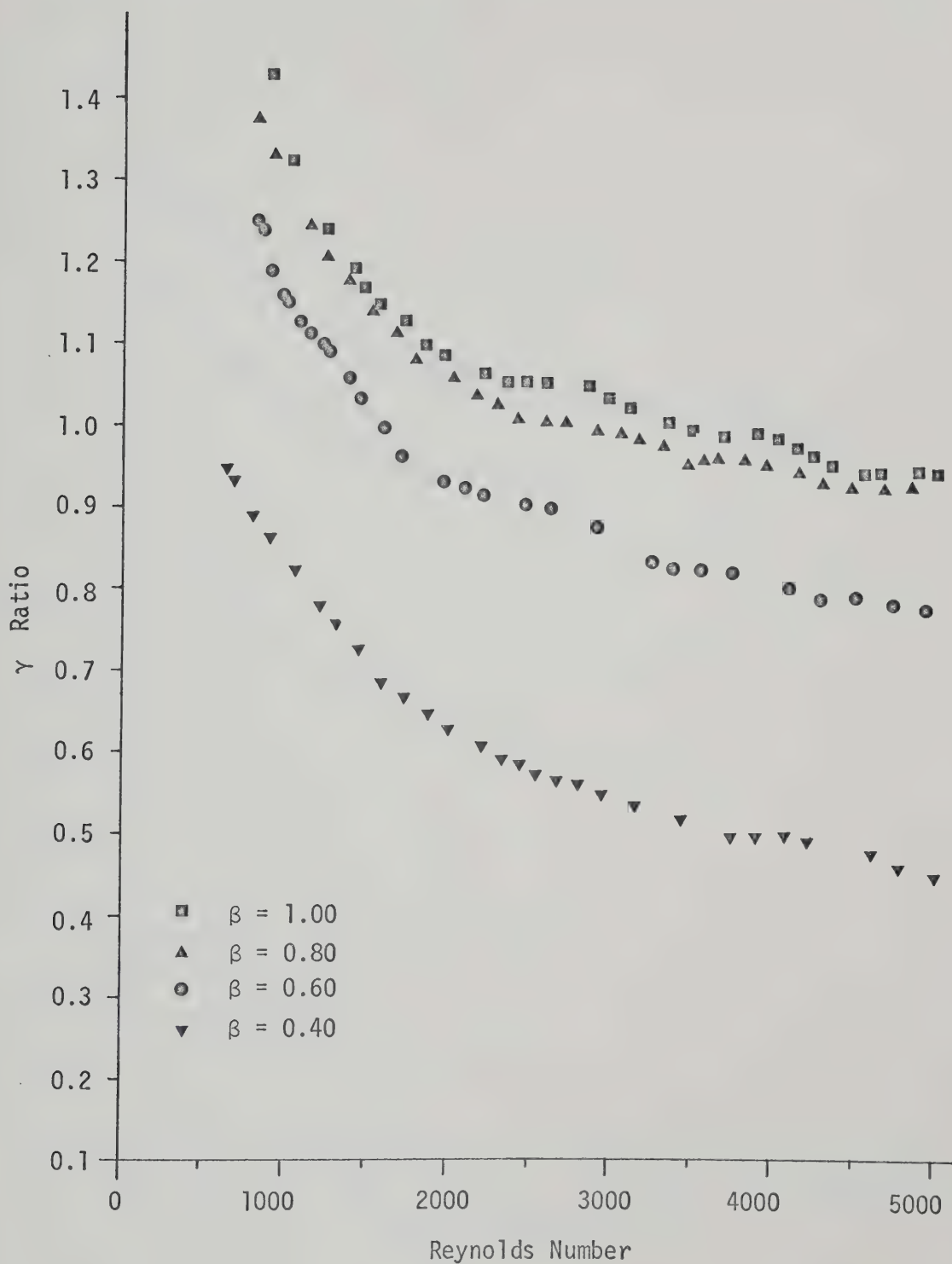


Figure 4.6 Round Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of β for $\alpha = 5/9$ ($\theta = 50^\circ$)

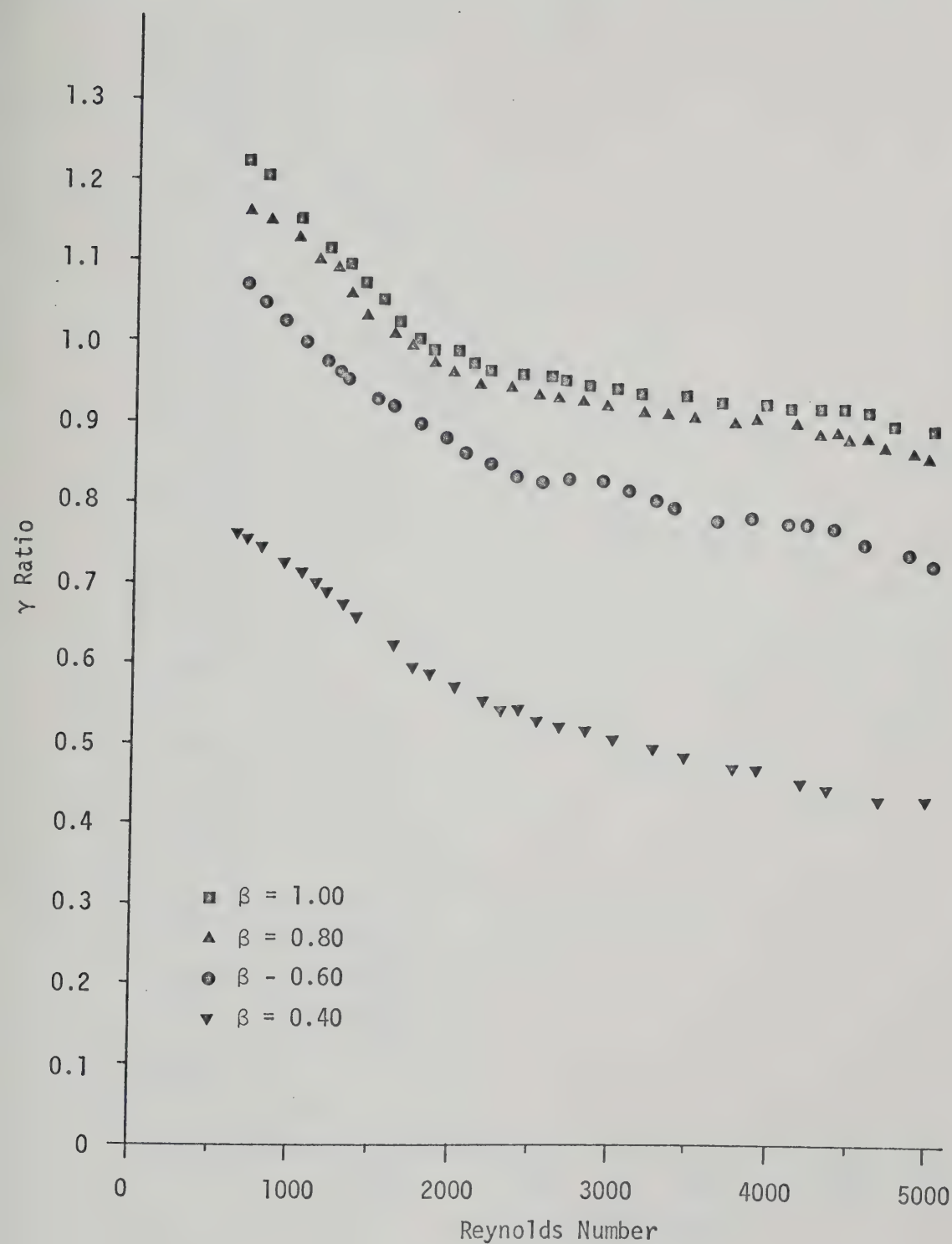


Figure 4.7 Round Edged Bifurcation
 Mass Flow Ratio γ versus Reynolds Number Re
 as a Function of β for $\alpha = 7/9$ ($\theta = 70^\circ$)

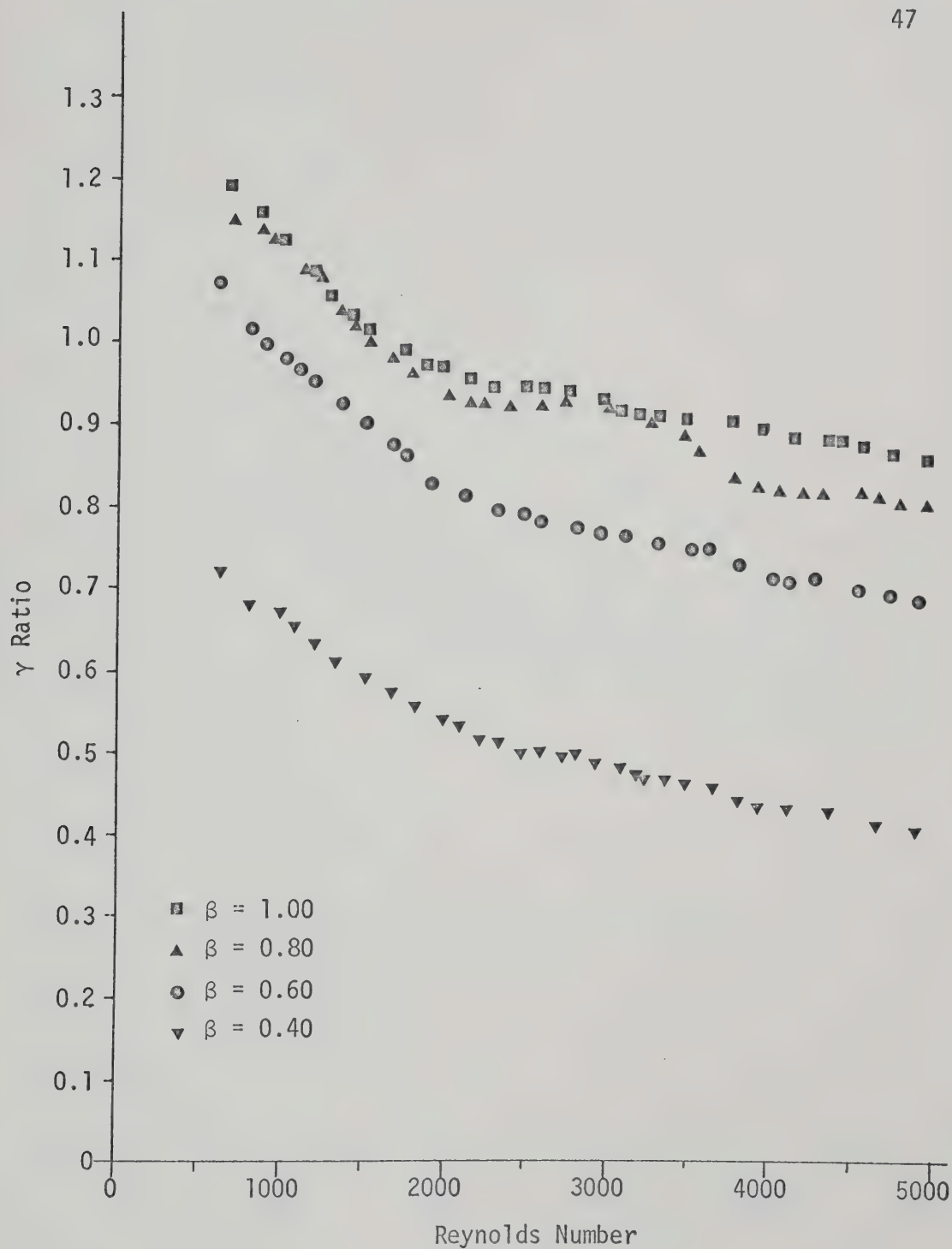


Figure 4.8 Round Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of β for $\alpha = 1$ ($\theta = 90^\circ$)

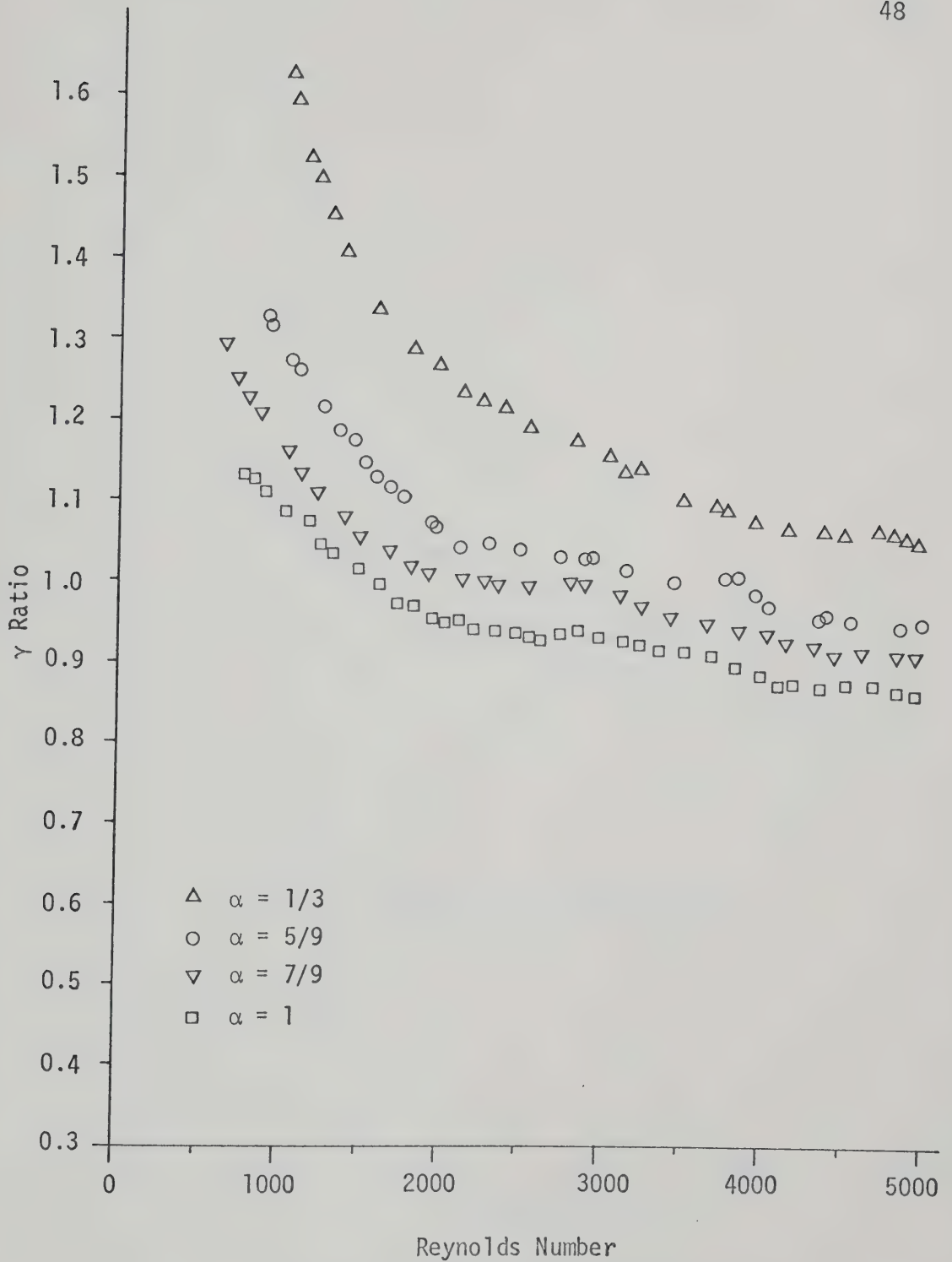


Figure 4.9 Sharp Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of α for $\beta = 1.00$

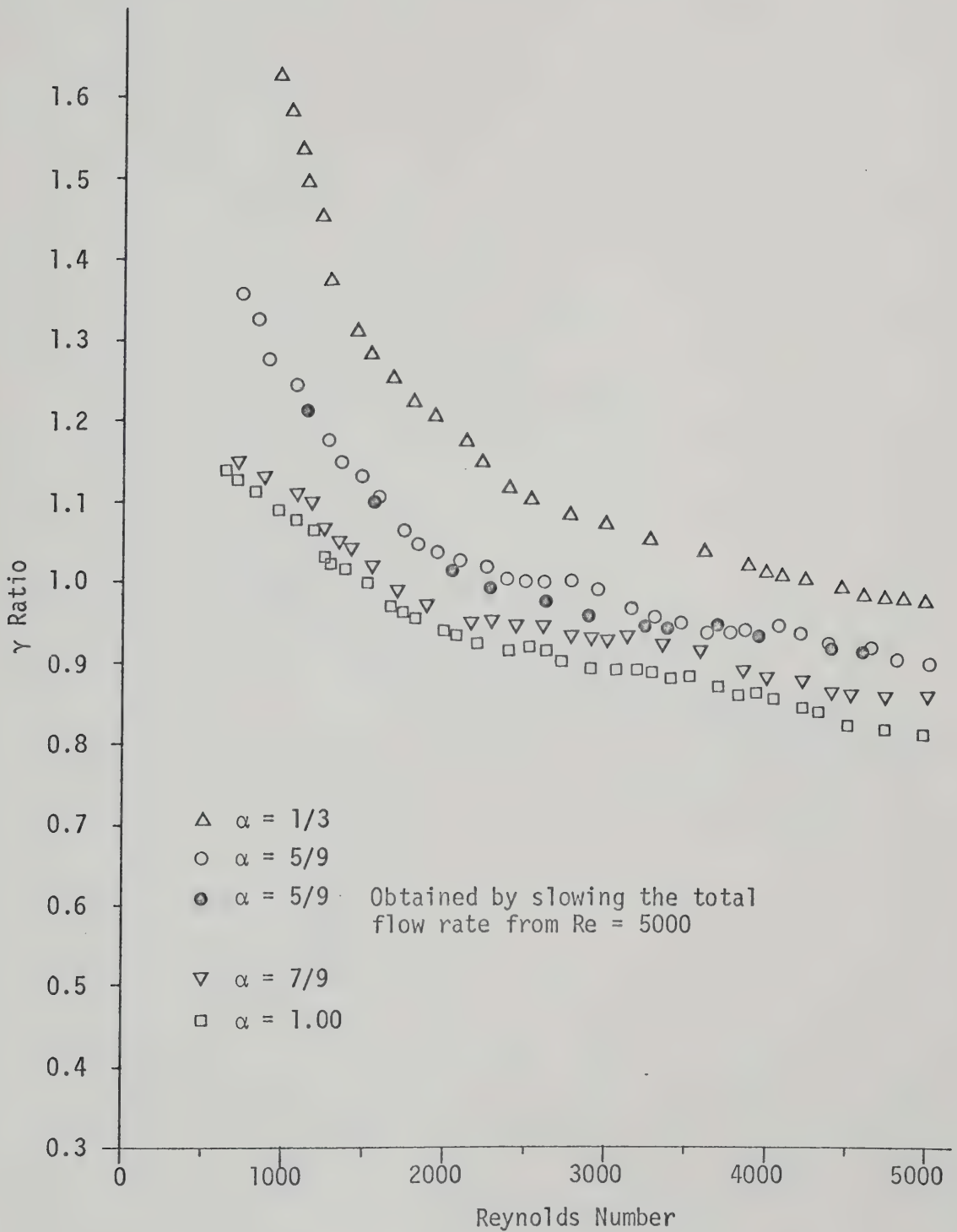


Figure 4.10 Sharp Edged Bifurcation
 Mass Flow Ratio γ versus Reynolds Number Re
 as a Function of α for $\beta = 0.80$

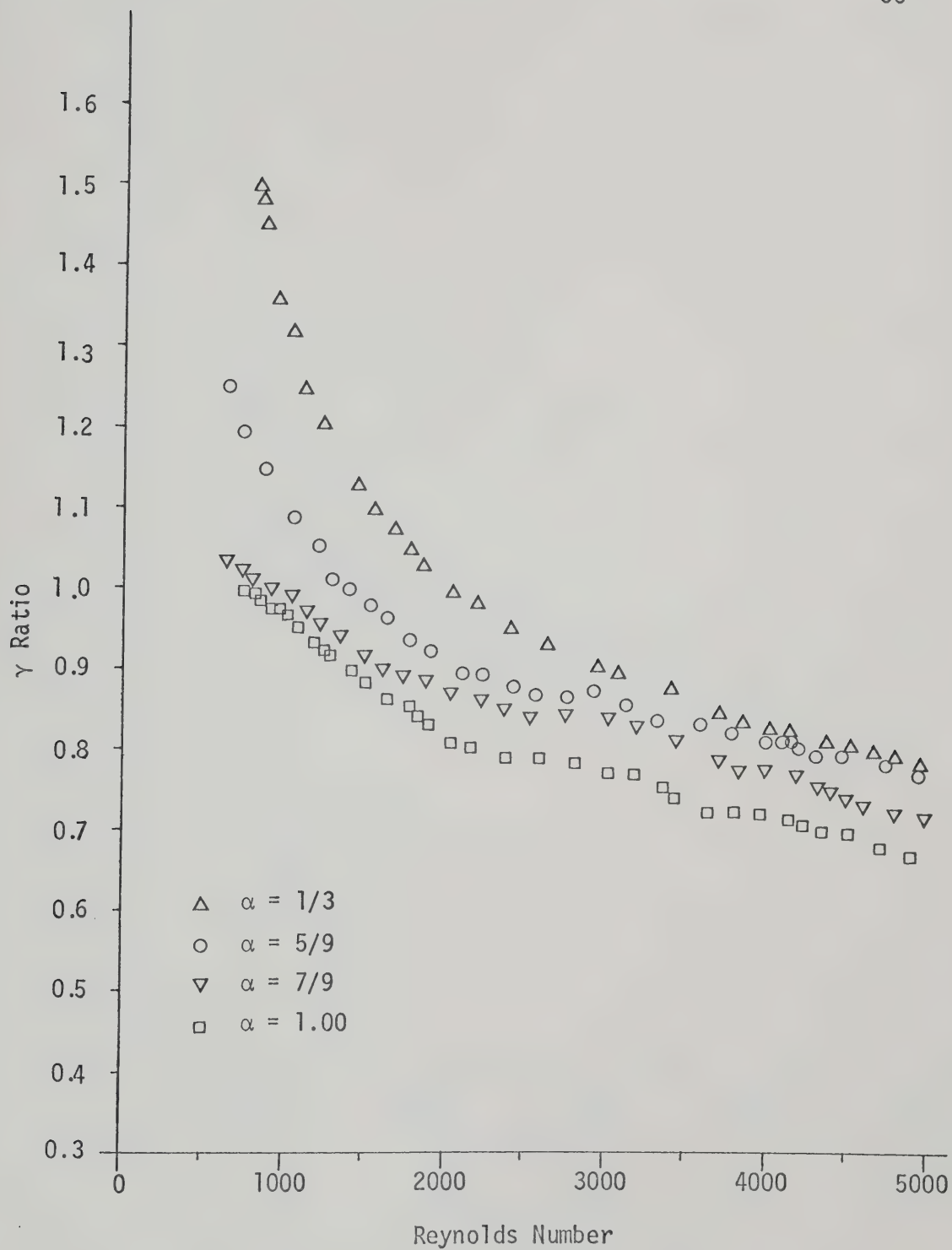


Figure 4.11 Sharp Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of α for $\beta = 0.60$

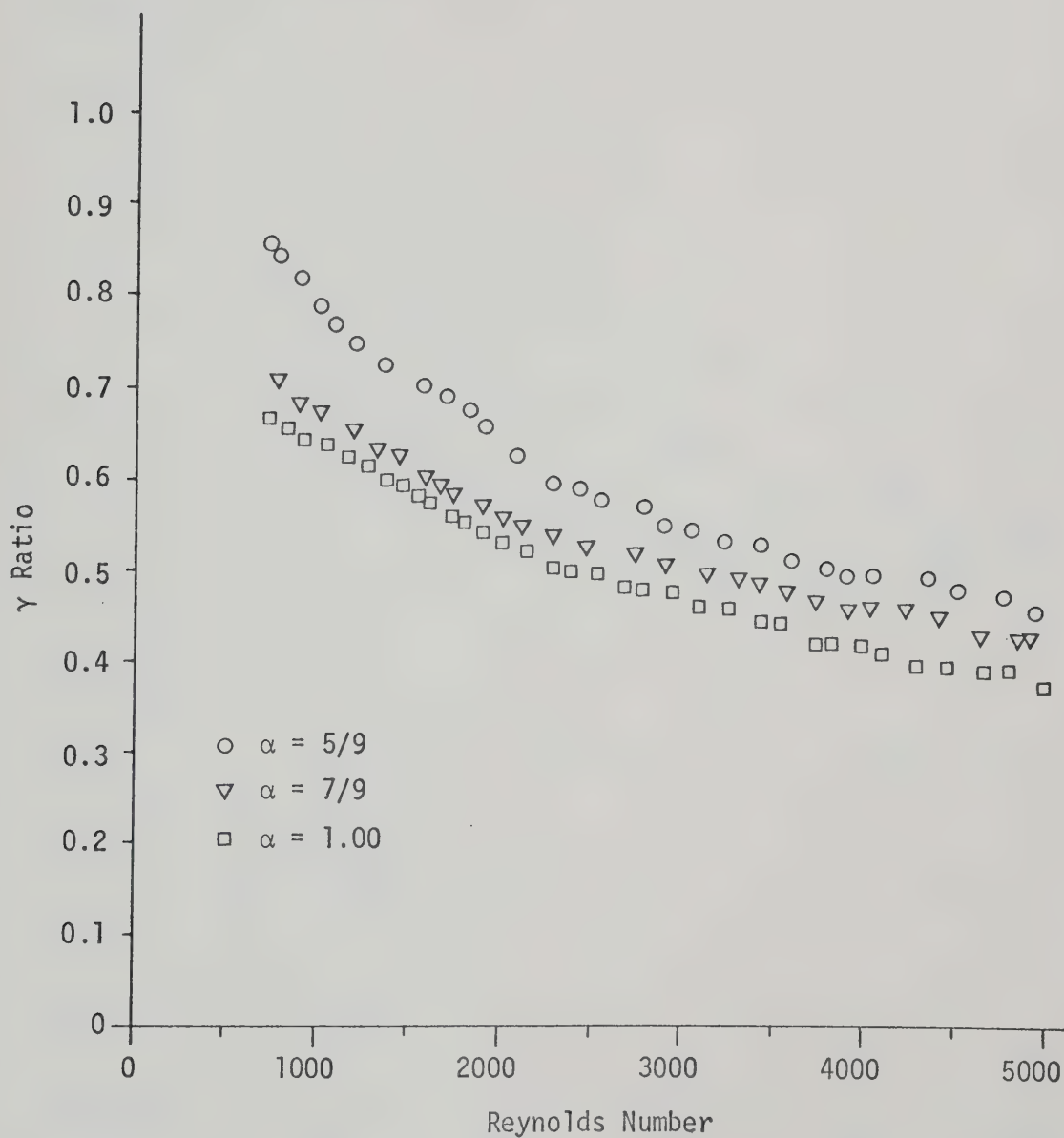


Figure 4.12 Sharp Edged Bifurcation
 Mass Flow Ratio γ versus Reynolds Number Re
 as a Function of α for $\beta = 0.40$

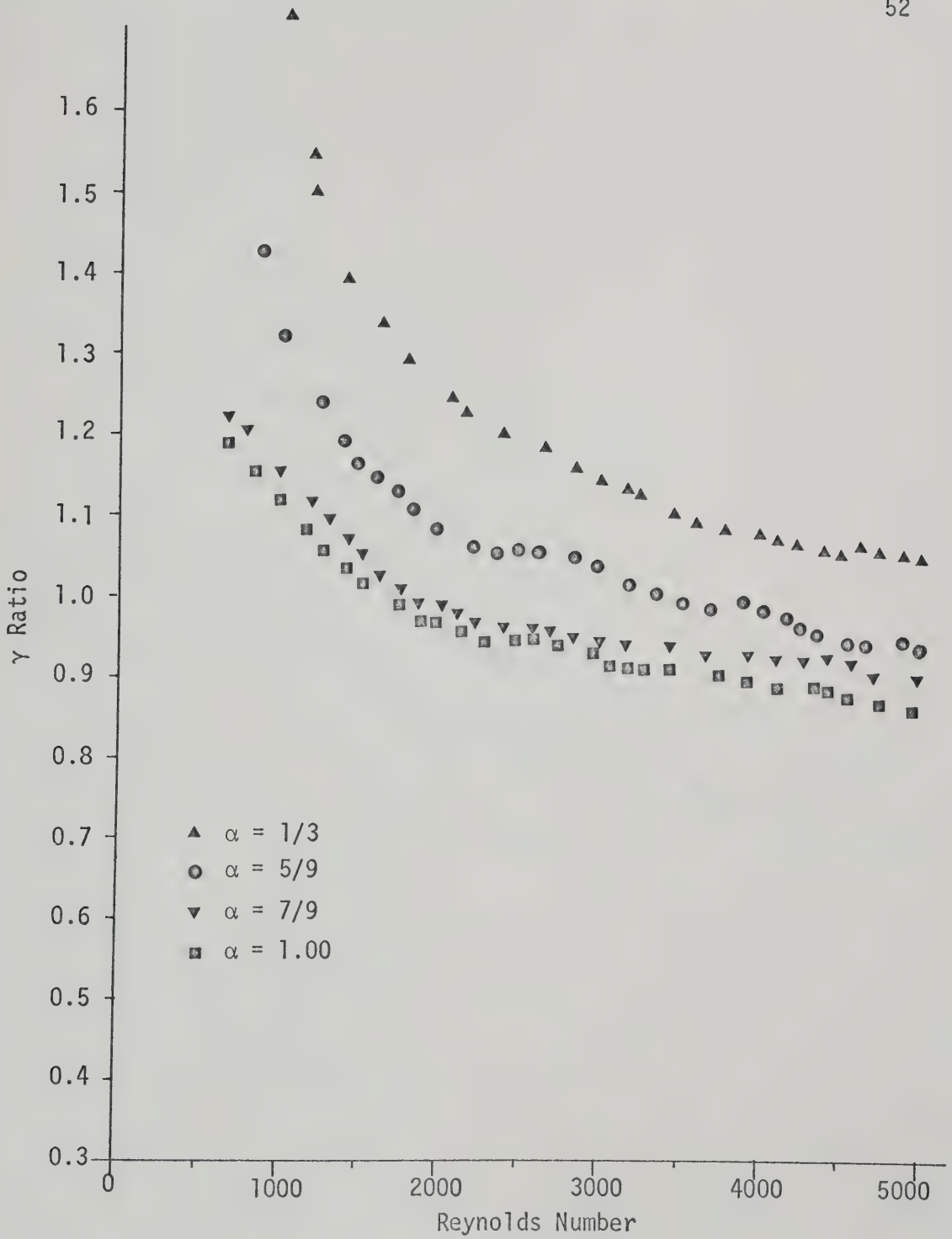


Figure 4.13 Round Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of α for $\beta = 1.00$

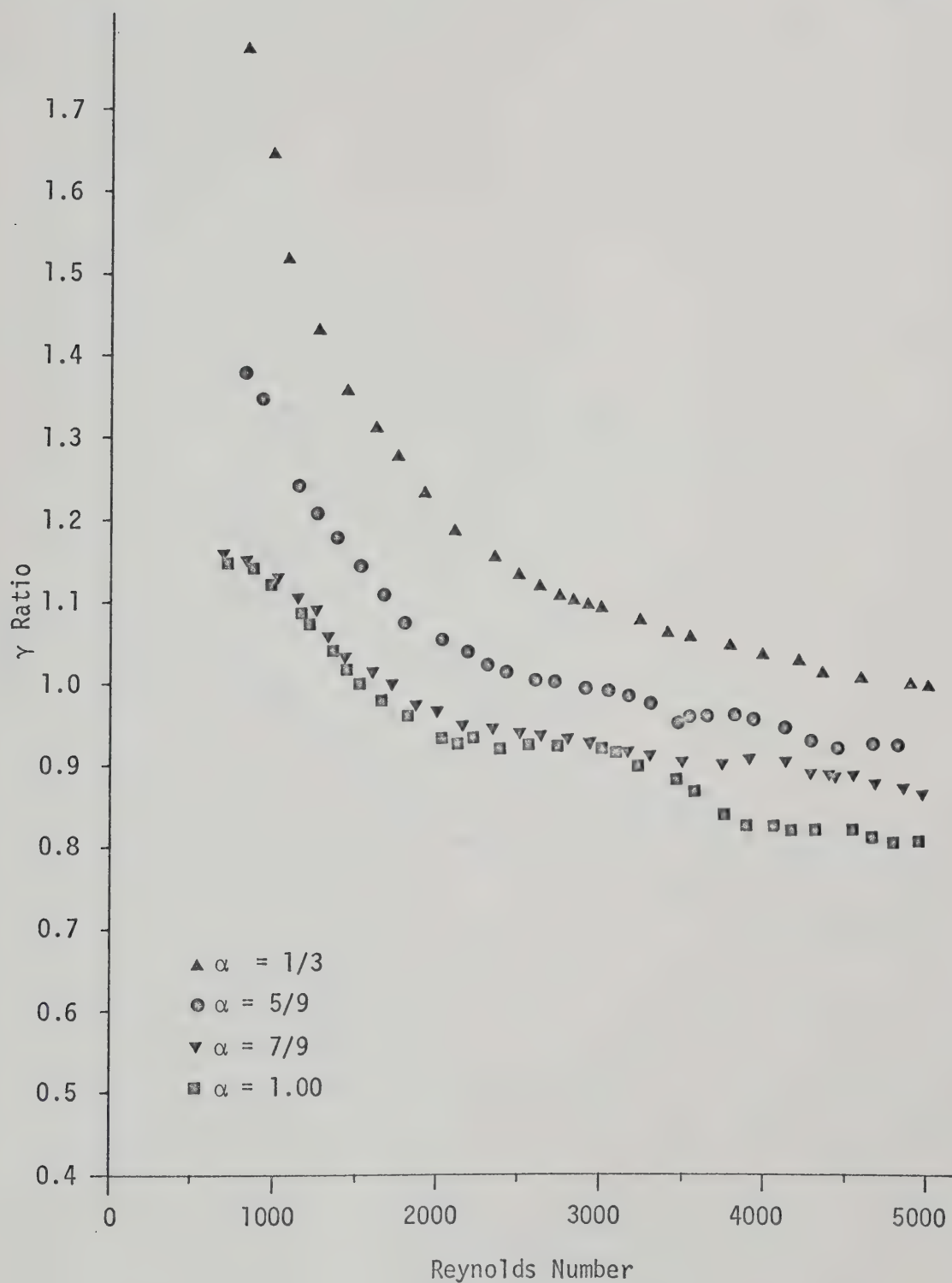


Figure 4.14 Round Edged Bifurcation
 Mass Flow Ratio γ versus Reynolds Number Re
 as a Function of α for $\beta = 0.80$

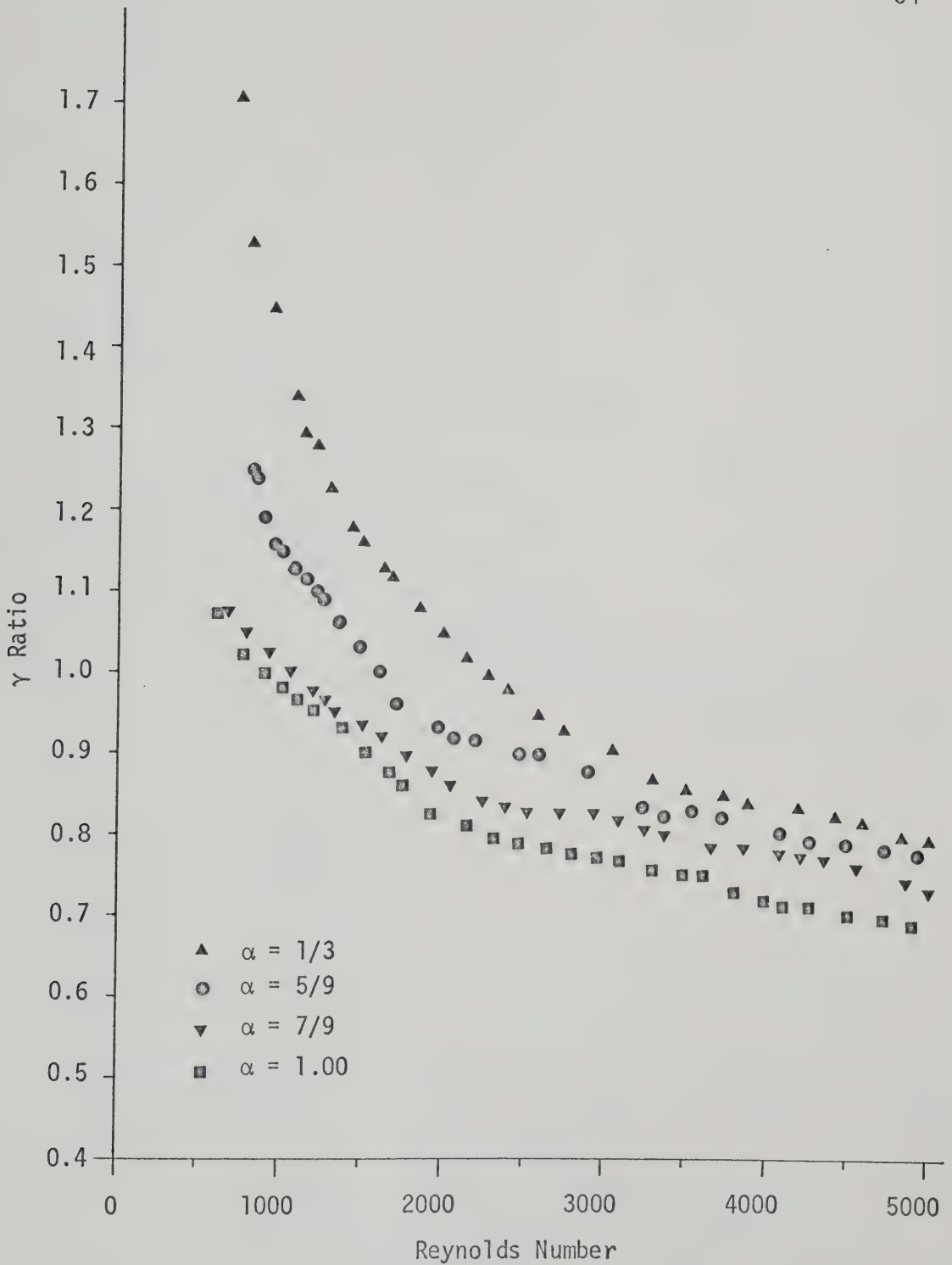


Figure 4.15 Round Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of α for $\beta = 0.60$

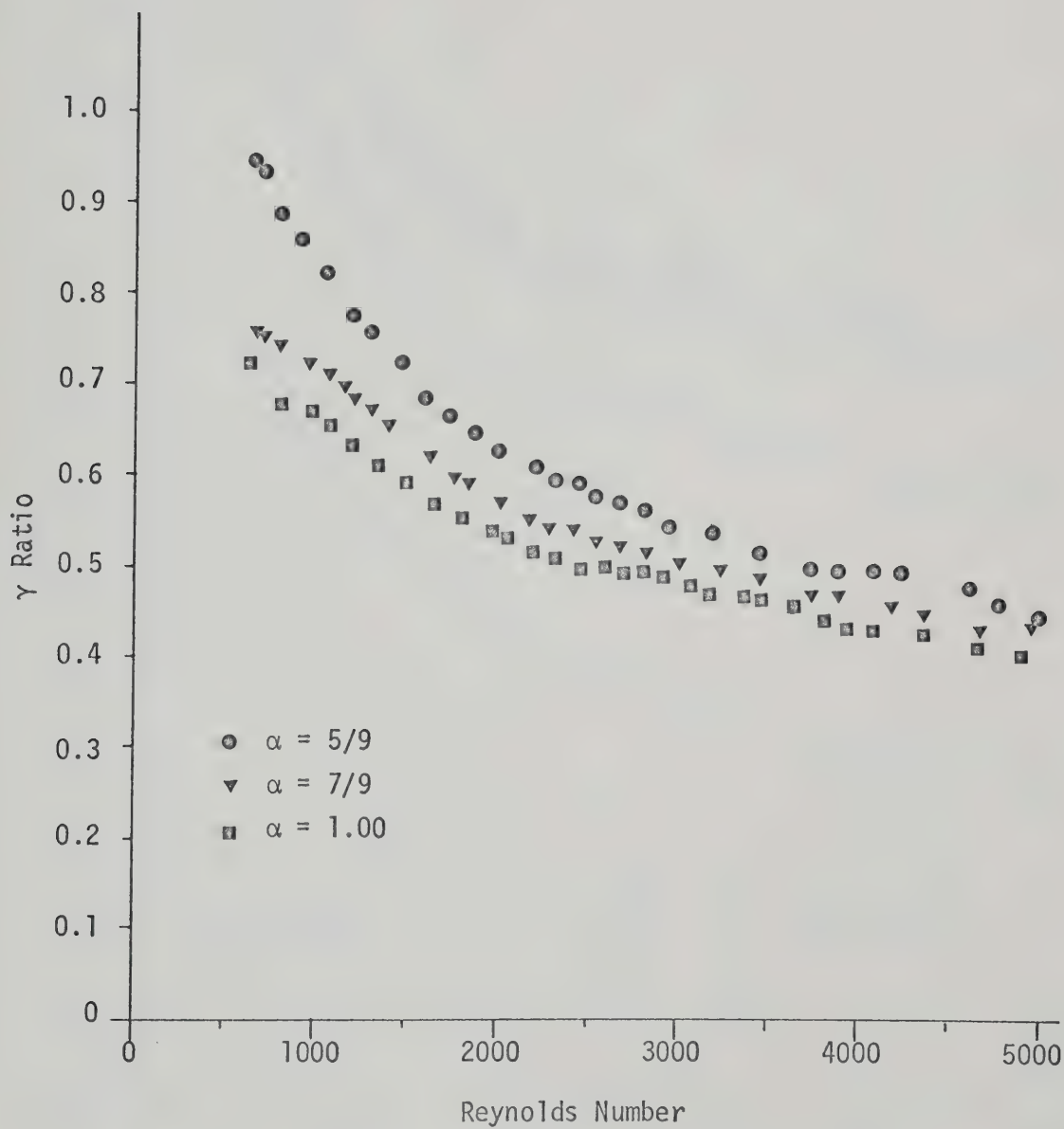


Figure 4.16 Round Edged Bifurcation
Mass Flow Ratio γ versus Reynolds Number Re
as a Function of α for $\beta = 0.40$

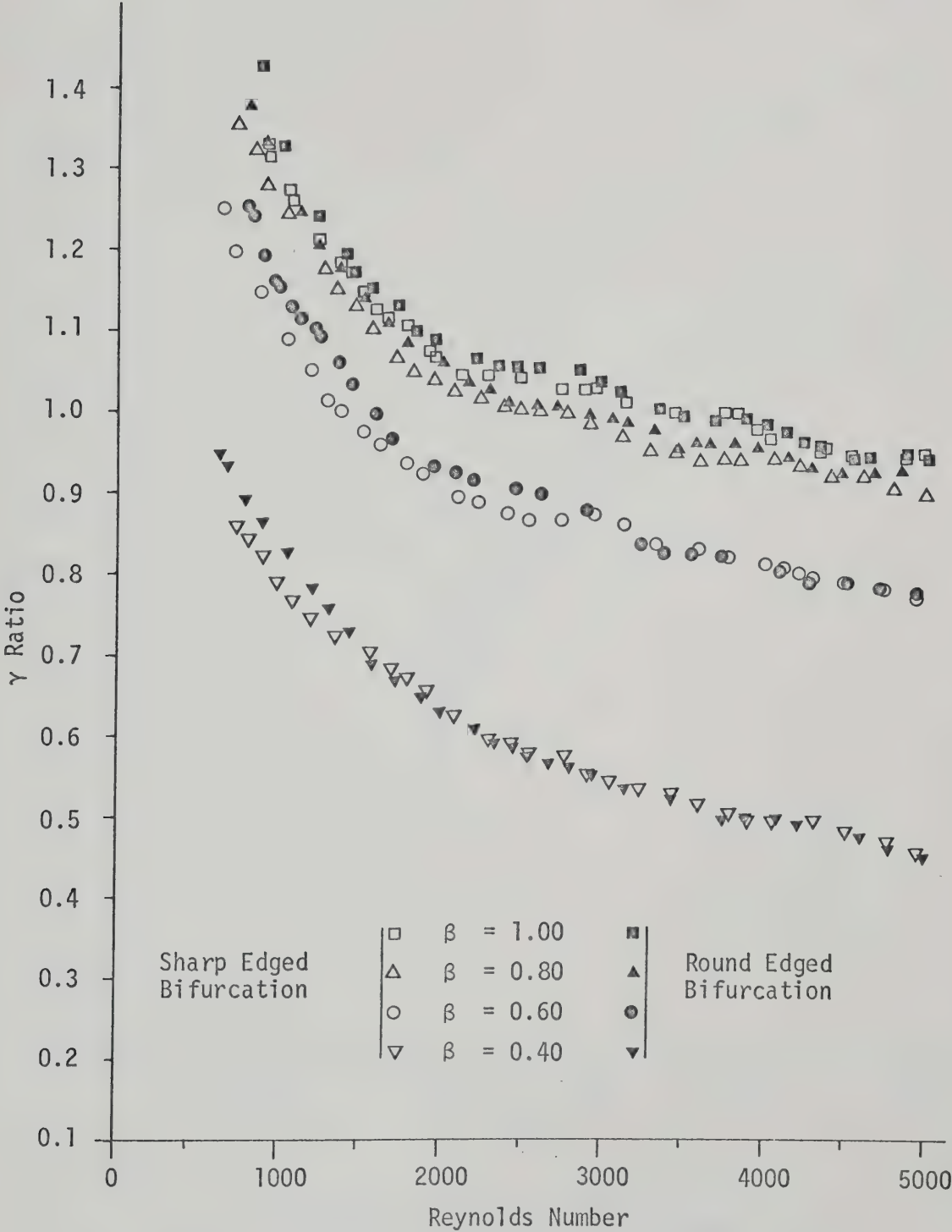


Figure 4.17 Comparison Between the Two Kinds of Bifurcations

Mass Flow Ratio γ versus Reynolds Number Re as a Function of β for $\alpha = 5/9$ ($\theta = 50^\circ$)

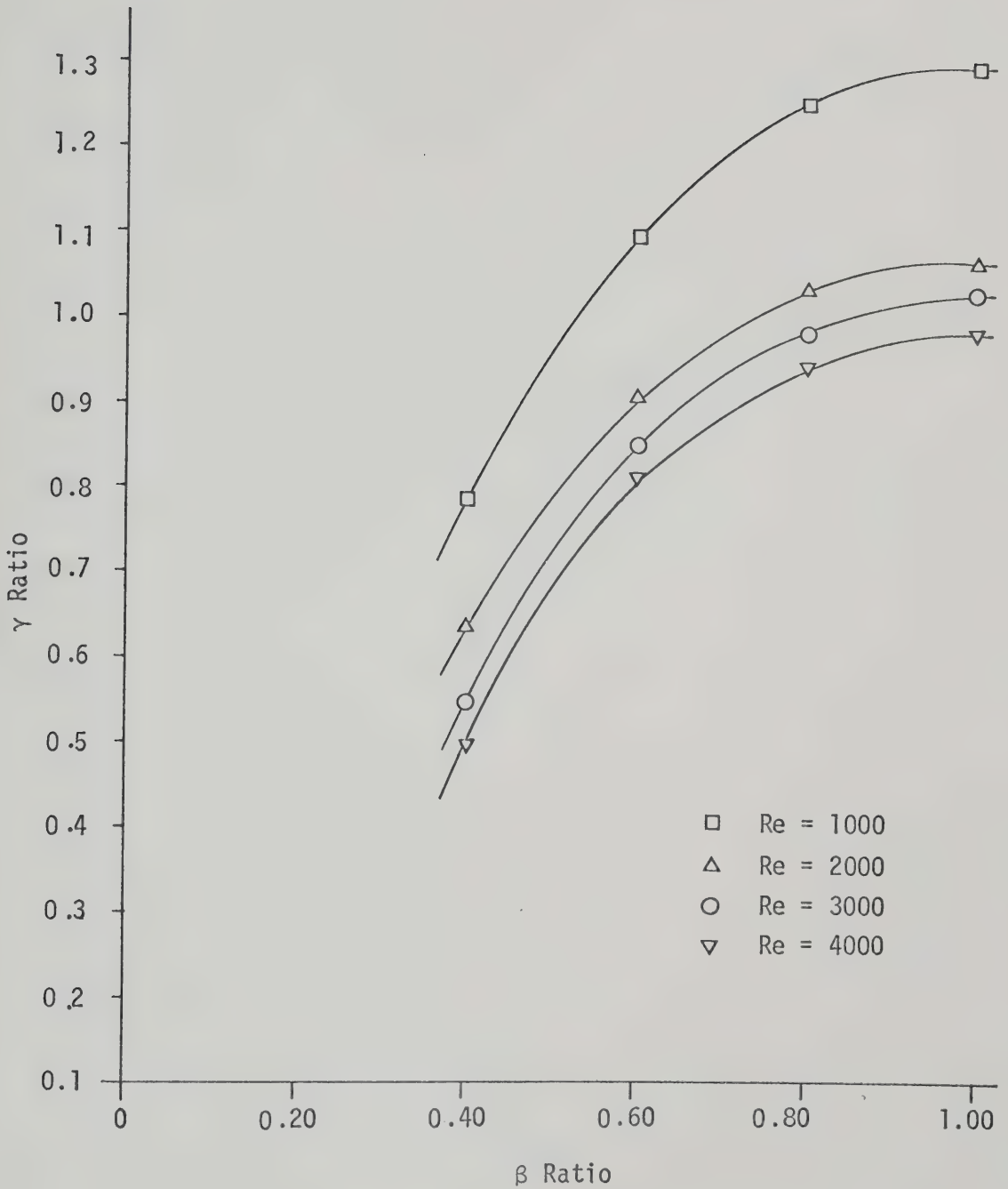


Figure 4.18 Sharp Edged Bifurcation
 Mass Flow Ratio γ versus Diameter Ratio β as a
 Function of Reynolds Number Re for
 $\alpha = 5/9$ ($\theta = 50^\circ$)

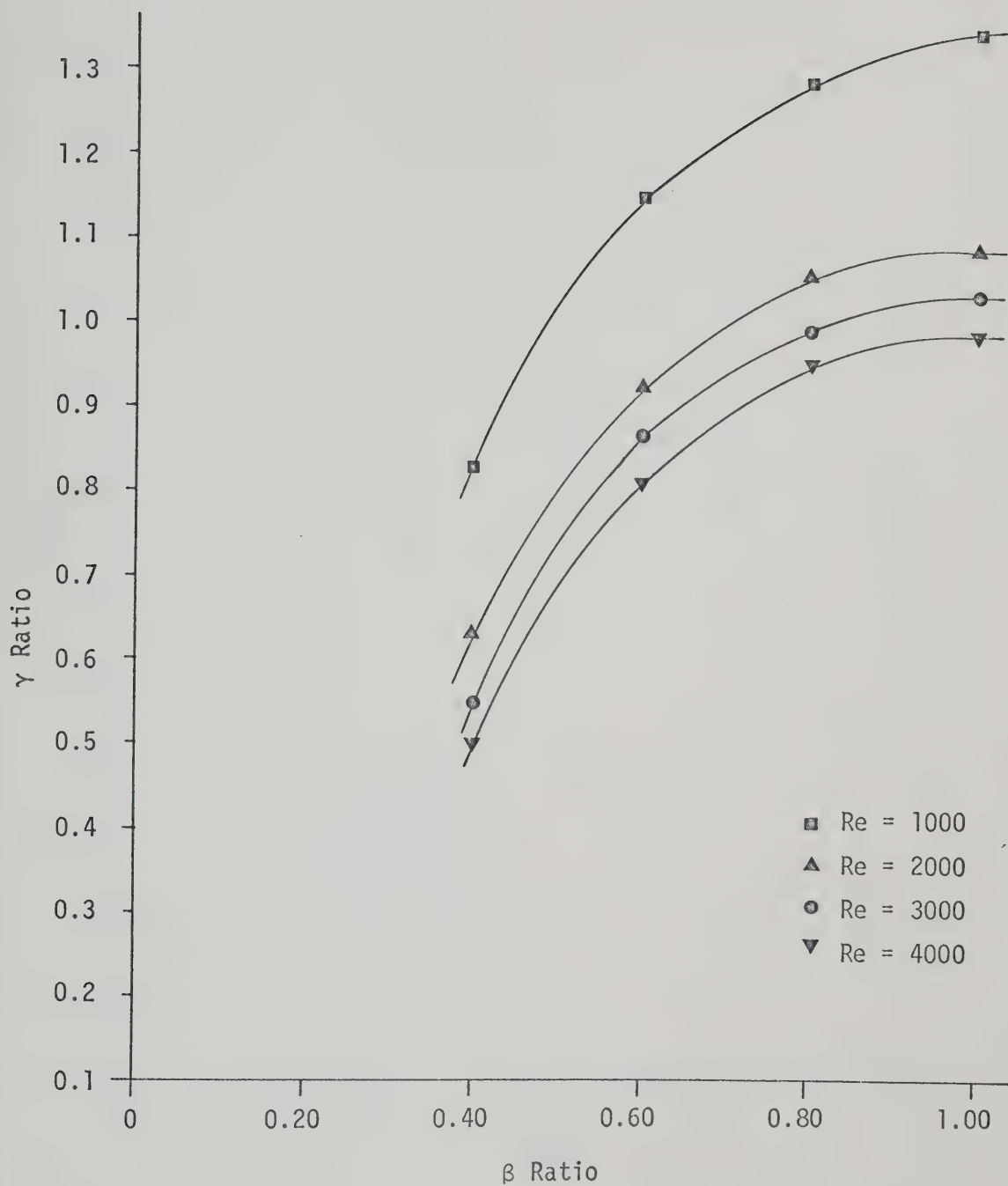


Figure 4.19 Round Edged Bifurcation
Mass Flow Ratio γ versus Diameter Ratio β as a
Function of Reynolds Number Re for
 $\alpha = 5/9$ ($\theta = 50^\circ$)

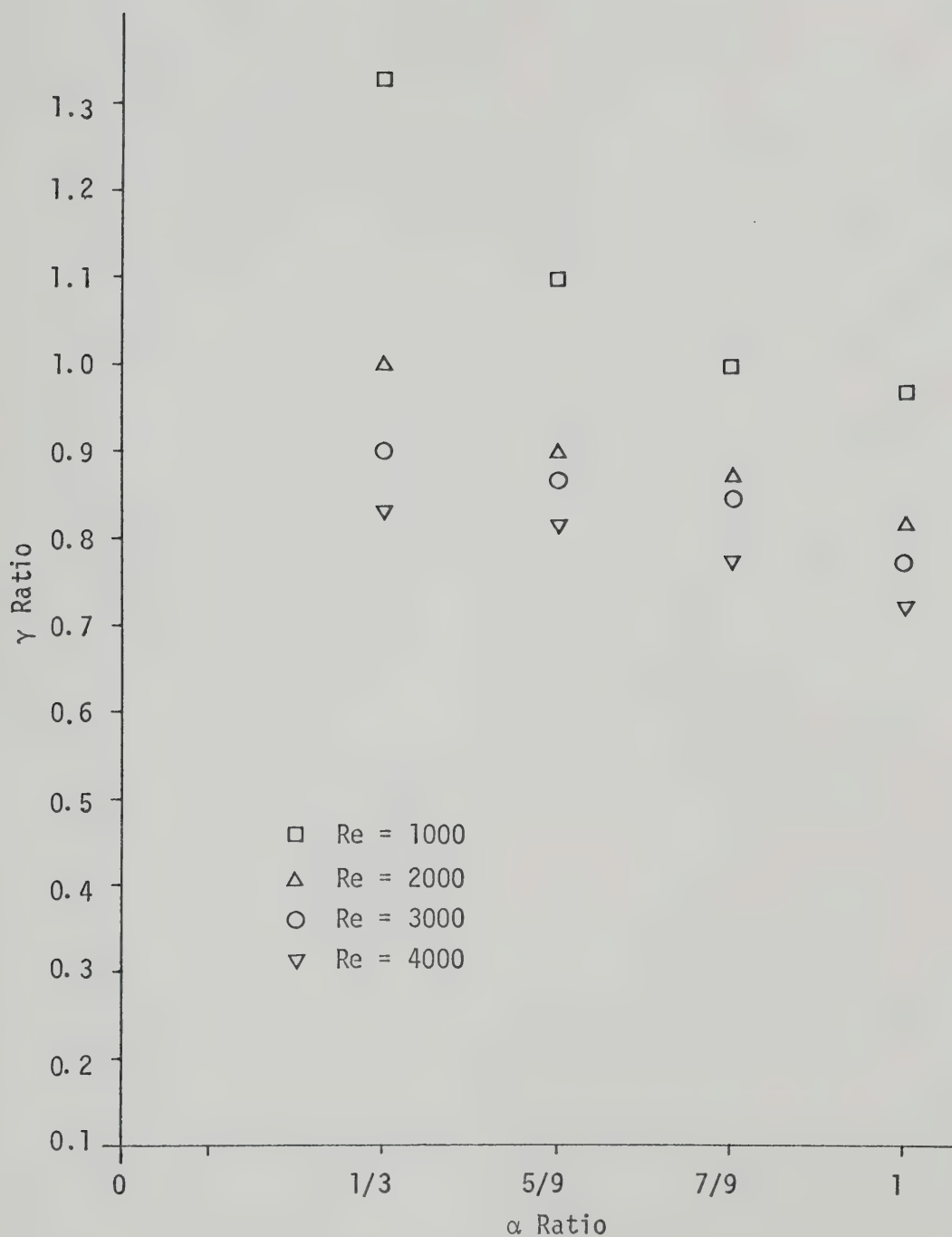


Figure 4.20 Sharp Edged Bifurcation
Mass Flow Ratio γ versus Non-Dimensional Angle α
as a Function of Reynolds Number for $\beta = 0.60$

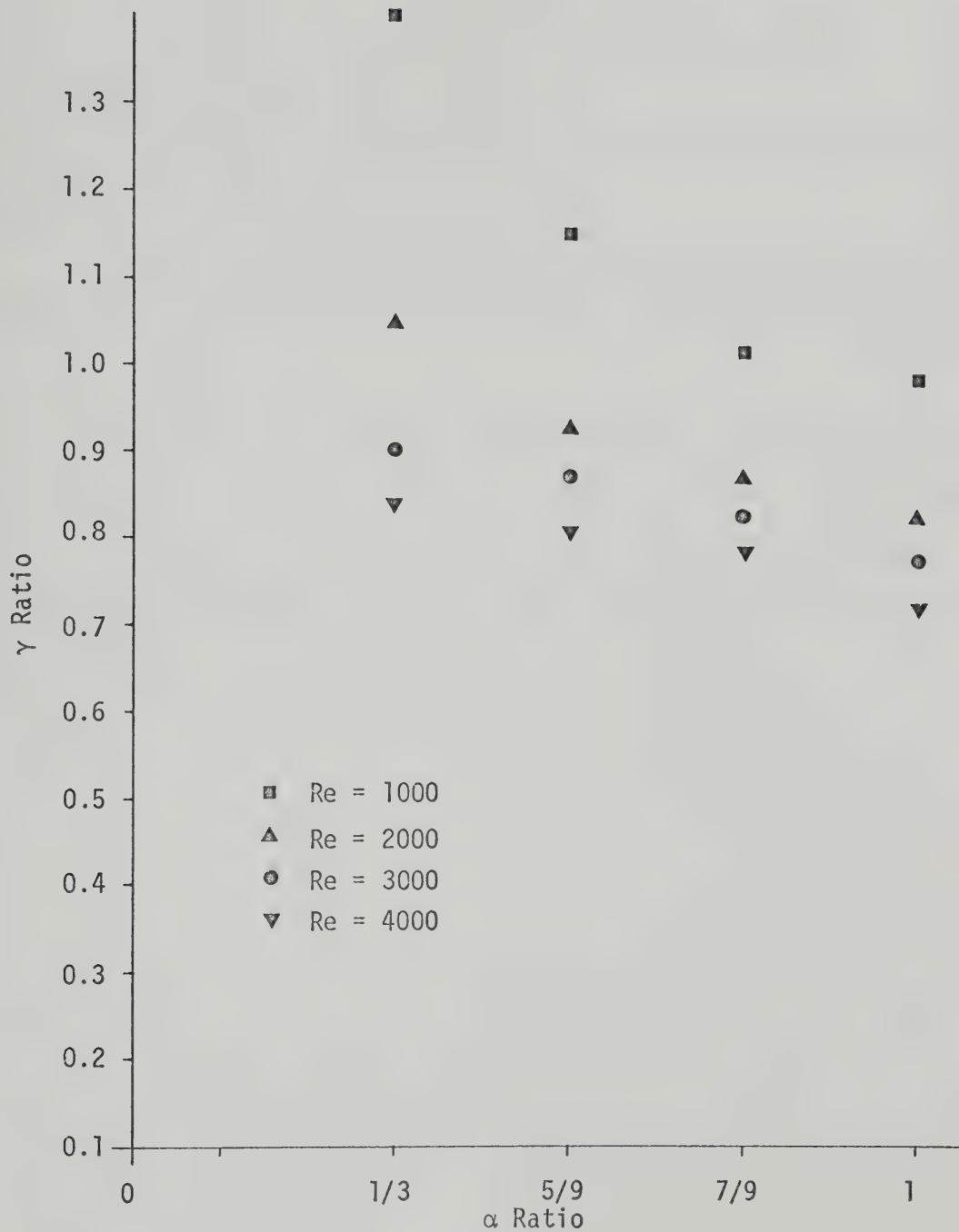


Figure 4.21 Round Edged Bifurcation
Mass Flow Ratio γ versus Non-Dimensional Angle α
as a Function of Reynolds Number for $\beta = 0.60$

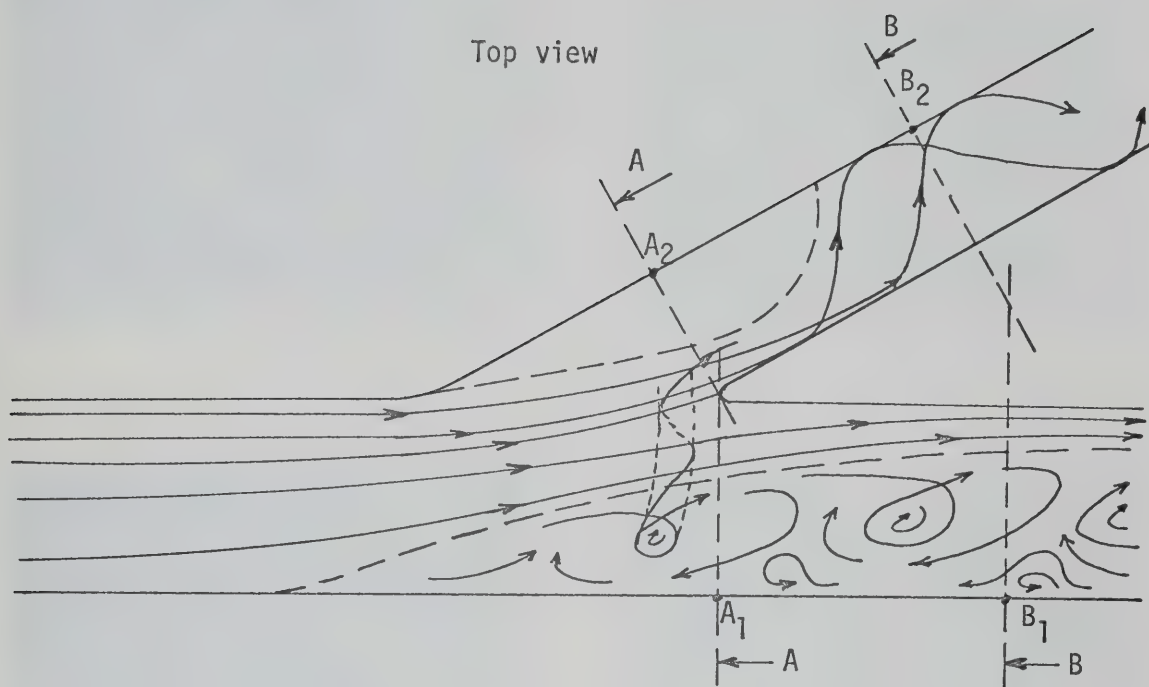
4.2.3 Description of the Flow

Figure 4.22 represents the flow in a bifurcation. This figure was sketched from photographs and observations.

Figures 4.23 and 4.24 show vortex formations in the separated region of the main branch and helicoidal flows in the side branch of a bifurcation, respectively.

The severity of the separated regions, for a given bifurcation, as a function of the Reynolds number, is given in Figures 4.25 and 4.26.

The dependence of the flow characteristics on the diameter ratio is shown in Figure 4.27, whereas their dependence on the branching-angle is studied in Figures 4.28 and 4.29.



Section A-A

Section B-B

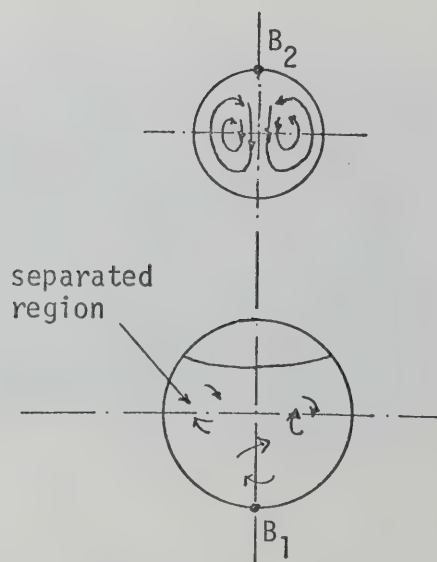
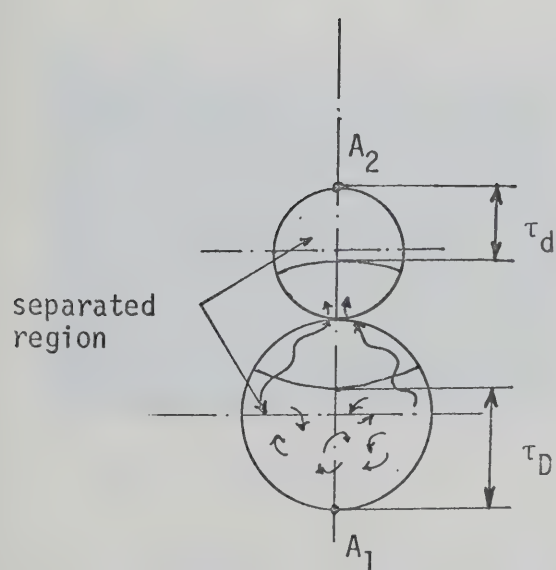


Figure 4.22 Physical Description of the Flow

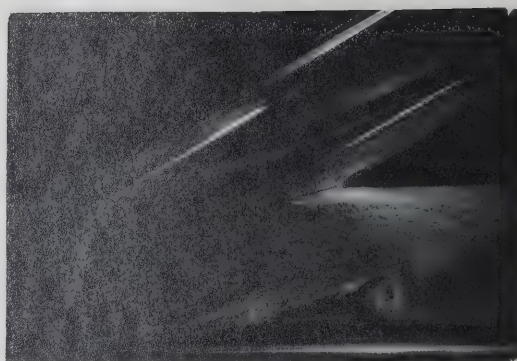
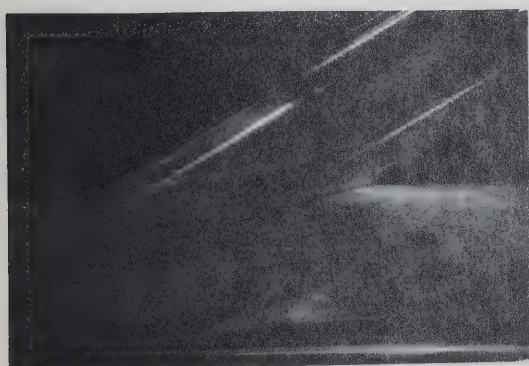
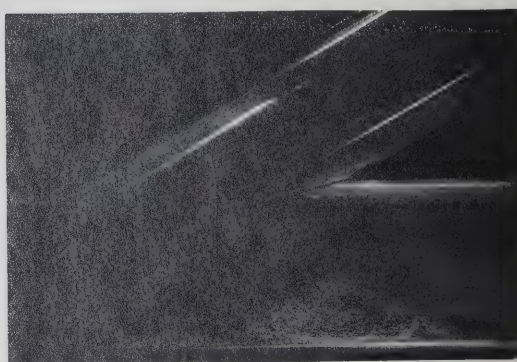
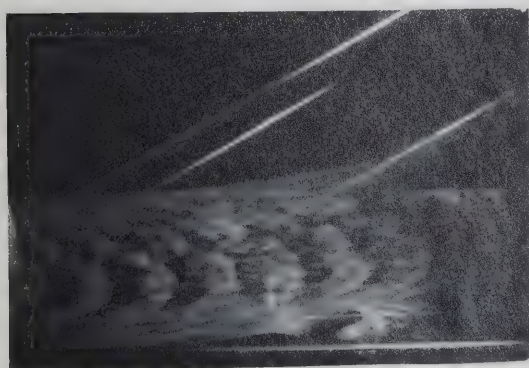
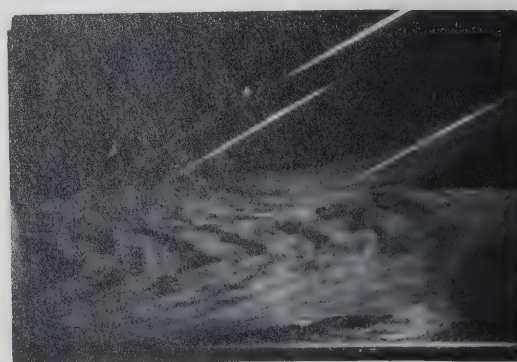
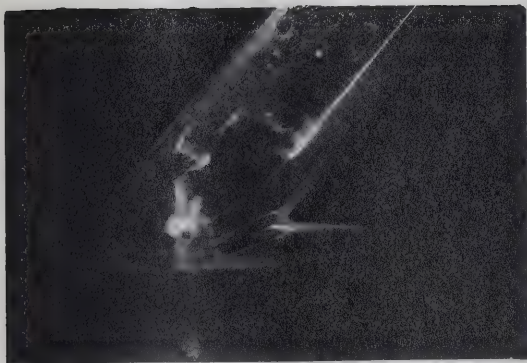
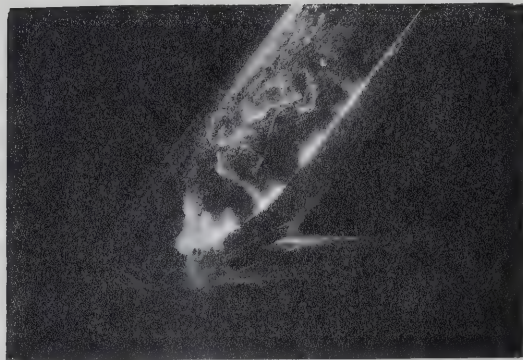
a) $\beta = 0.80$ $Re = 1130$ b) $\beta = 0.80$ $Re = 1100$ c) $\beta = 0.80$ $Re = 1180$ d) $\beta = 0.80$ $Re = 1410$ e) $\beta = 1.00$ $Re = 1520$ f) $\beta = 1.00$ $Re = 1520$

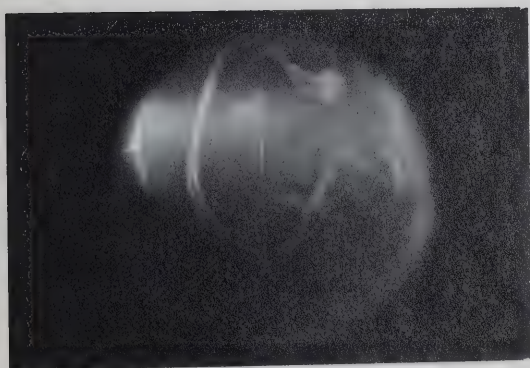
Figure 4.23 Vortex Formations in the Main Branch for a
Sharp Lined Bifurcation with
 $\alpha = 1/3$ ($\theta = 30^\circ$)



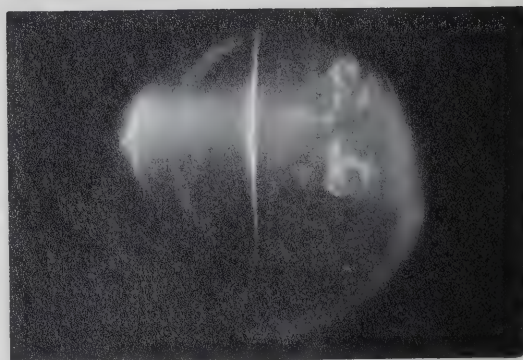
(a)



(b)



(c)



(d)



(e)



(f)

Figure 4.24 Double-Helical Flow in the Side Branch, for a Sharp Edged Bifurcation with $\beta = 0.00$ and $\alpha = 1/3$ ($\theta = 30^\circ$) $Re = 2100$

- a and b are viewed from the top
- c to f are viewed through the side-branch constant head tank

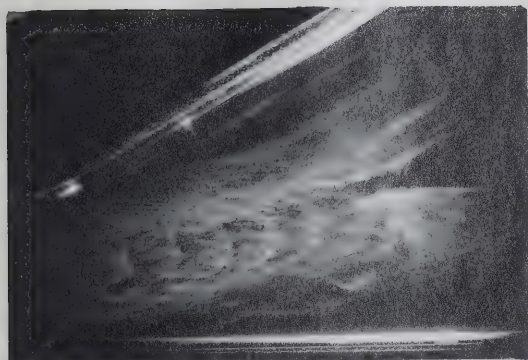
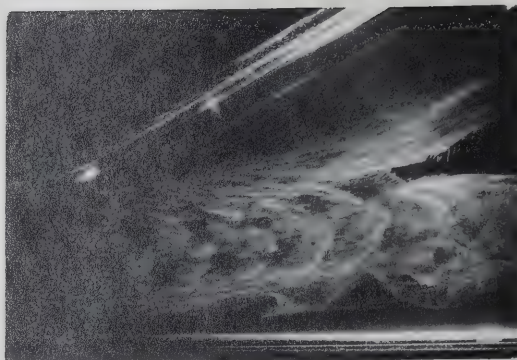
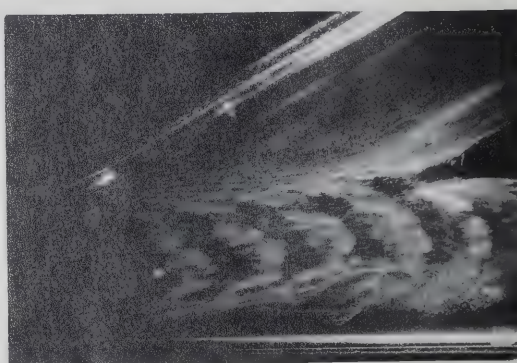
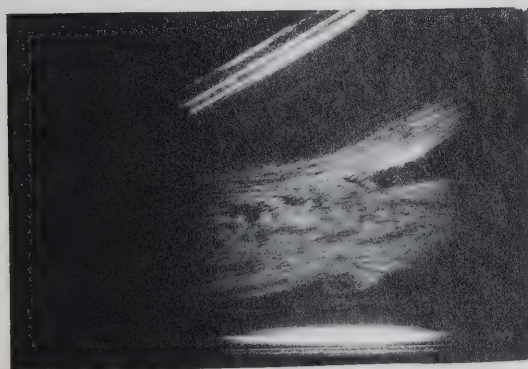
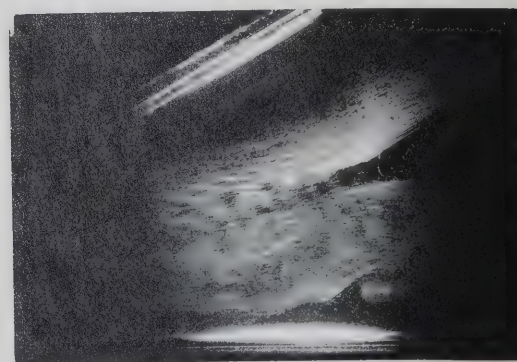
a) $Re = 1400$ b) $Re = 1530$ c) $Re = 1840$ d) $Re = 1970$ e) $Re = 2060$ f) $Re = 2060$

Figure 4.25 Flow Characteristics for Various Reynolds Numbers:
Round Edged Bifurcation with
 $\beta = 1.00$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

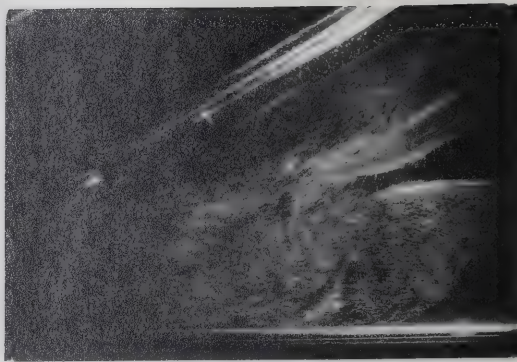
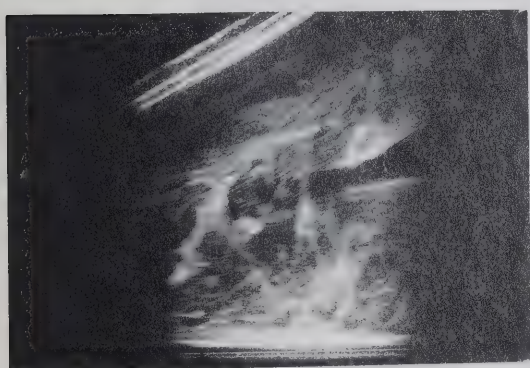
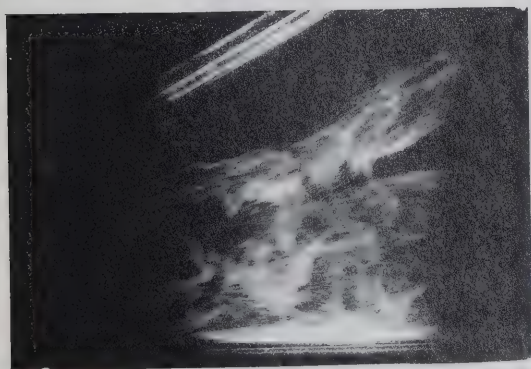
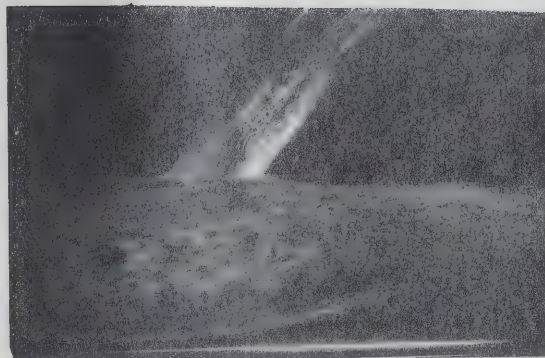
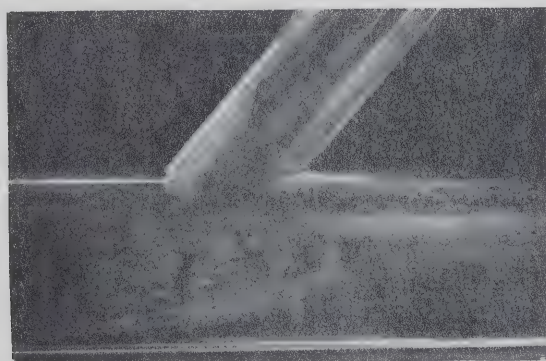
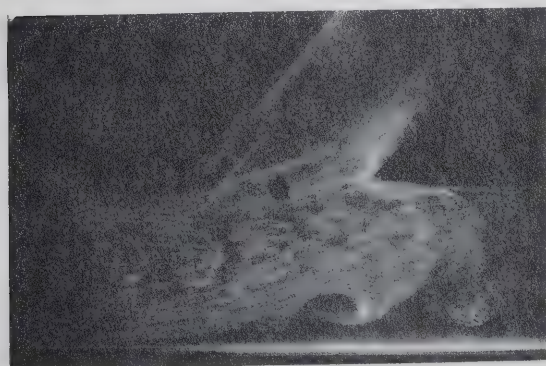
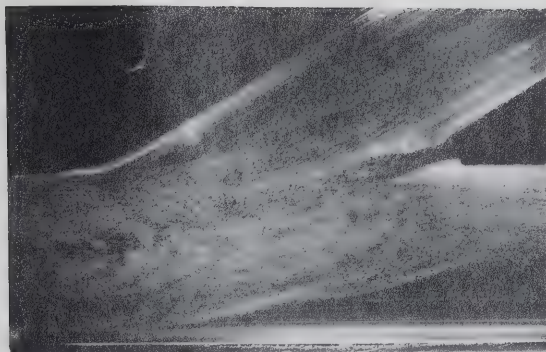
a) $Re = 2350$ b) $Re = 2730$ c) $Re = 3510$ d) $Re = 3510$ e) $Re = 4000$ f) $Re = 4060$

Figure 4.26 Flow Characteristics for Various Reynolds Numbers
 Round Edged Bifurcation with
 $\beta = 1.00$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

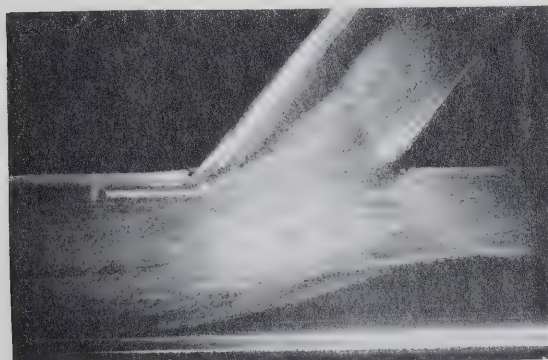
a) $\beta = 0.40$ $Re = 1650$ b) $\beta = 0.60$ $Re = 1600$ c) $\beta = 0.80$ $Re = 1610$ d) $\beta = 1.00$ $Re = 1590$

Sharp Edged Bifurcation
with $\alpha = 5/9$ ($\theta = 50^\circ$)

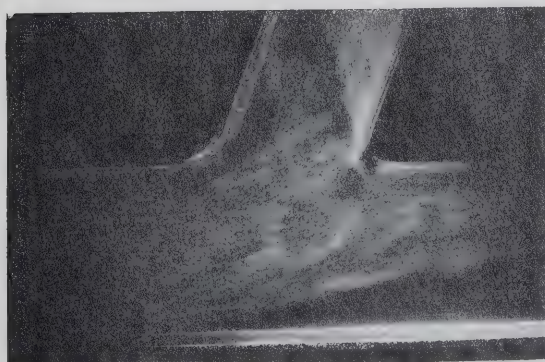
Figure 4.27 Flow Characteristics for Various
Diameter Ratios



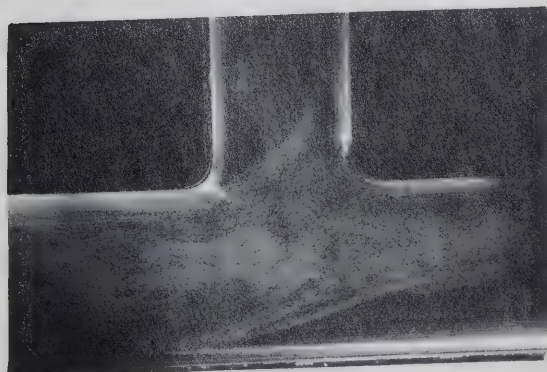
$$a) \quad \alpha = \frac{1}{3} \quad Re = 1160$$



$$b) \quad \alpha = \frac{5}{9} \quad Re = 1160$$



$$c) \quad \alpha = \frac{7}{9} \quad Re = 1170$$



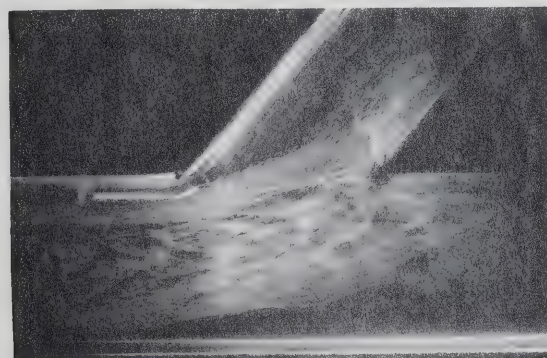
$$d) \quad \alpha = 1 \quad Re = 1150$$

Found Edged Bifurcation
with $\beta = 0.80$

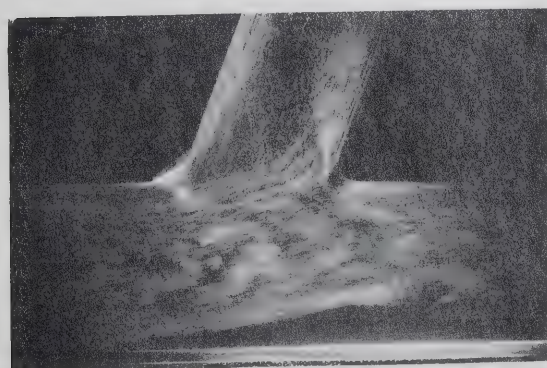
Figure 4.28 Flow Characteristics for Various
Angles of Branching



a) $\alpha = \frac{1}{3}$ $Re = 1740$



b) $\alpha = \frac{5}{9}$ $Re = 1740$



c) $\alpha = \frac{7}{9}$ $Re = 1760$



d) $\alpha = 1$ $Re = 1750$

Round Edged Bifurcation
with $\beta = 0.80$

Figure 4.29 Flow Characteristics for Various
Angles of Branching

4.3 Observations and Discussions

4.3.1 Mass Flow Ratio, γ

The study of the curves given in Figures 4.1 through 4.17 leads to the following conclusions:

1. The mass flow ratio, γ , for a given bifurcation, is a decreasing function of the Reynolds number. Since an increase of the Reynolds number increases the fluid momentum upstream of the bifurcation, less fluid will flow through the side branch.

2. For a Reynolds number between 2000 and 3000, the value of the mass flow ratio, γ , for a given bifurcation, depends on the flow history. Figure 4.10 shows that γ is higher when the flow is increased from laminar to turbulent than when it is decreased from turbulent to laminar. This constitutes the transition region of the bifurcation.

3. A given increase of the diameter ratio, β , with everything else unchanged, will result in an increase of the mass flow ratio, γ . This increase is a decreasing function of β , as also shown in Figures 4.18 and 4.19.

4. A given decrease of the angle of branching, θ , with everything else unchanged, will result in an increase of the mass flow ratio, γ . This increase is a decreasing function of α ($\alpha = \theta/90^\circ$) as also shown in Figure 4.20 and 4.21.

5. The previous conclusions are valid whether the bifurcation has sharp or rounded inside edges. The general behaviour of the flow is then the same for the two kinds of bifurcations. However, for a Reynolds number less than 2000, the mass flow ratio, γ , everything else

being unchanged, is slightly higher for a round edged than for a sharp edged bifurcation (see Figure 4.17). This, consequently, applies to the physiological range of Reynolds number (see paragraph 3.1) which is:

$$1300 < Re < 2040 \quad (4.1)$$

This difference is probably due to the fact that the point of separation in the side branch is pushed downstream into the side branch for a round edged bifurcation with respect to the corresponding sharp edged bifurcation.

4.3.2 Separation Regions

Photographs and observations clearly prove the existence of two interdependent separation regions: one in the main branch and one in the side branch. If we defined their thicknesses by τ_D and τ_d , respectively, according to Figure 4.22, Figures 4.25 through 4.29 show that, whenever τ_D increases, τ_d decreases and vice versa. A growth of the separated region in the main branch will then restrict the flow in the main branch and will, consequently, increase γ .

The thickness, τ_D , of the separated region in the main branch increases when:

1. the Reynolds number decreases (Figures 4.25 and 4.26)
2. the diameter ratio, β , increases (Figure 4.27)
3. the angle of branching decreases (Figures 4.28 and 4.29).

This is then the physical explanation of the previous conclusions on the mass flow ratio, γ .

Inside the separated region in the main branch, back flows generate vortices, as shown in Figures 4.22, 4.23 and 4.24 (e and f). These vortices are not limited to the diametrical plane of the bifurcation. In fact, they have been observed to present a banana-shape, which enables particles to be transferred from the main branch into the side branch, as sketched in Figure 4.22.

Inside the side branch, the flow becomes double-helicoidal (see Figures 4.22 and 4.24). This kind of flow is typical of flows in curved pipes, as explained by H. Schlichting.¹⁷

4.3.3 Limitations of the Apparatus

The apparatus limits the accuracy of the results to approximately $\pm 2\%$. These results have been found to be reproducible within $\pm 2\%$ for Reynolds numbers greater than 1000 (see Figure 4.5). This reproducibility is directly proportional to the care taken to obtain precisely the same level for the two constant head tanks, since the flow through the bifurcation is highly dependent on the resistance in each branch.

For Reynolds numbers less than 1000 the reproducibility becomes questionable, due to surface tension effects in the constant head tanks. Also, below that Reynolds number ($Re = 1000$), the visualization technique becomes useless, due to the growing importance of the buoyancy effect on the hydrogen bubbles.

Finally, from the observations and the photographs, no definitive conclusion can be drawn as far as the location of the separation point in the main branch is concerned.

CHAPTER V

CONCLUSIONS

5.1 Summary

In conclusion , for the physiological range of Reynolds number, the experimental findings can be summarized as follows:

1. The mass flow ratio, γ , is a decreasing function of the entrance Reynolds number.
2. The mass flow ratio, γ , is an increasing function of the diameter ratio, β .
3. The mass flow ratio, γ , is a decreasing function of the angle of branching, θ .
4. The mass flow ratio, γ , is slightly higher for a round edged bifurcation than for the corresponding sharp edged bifurcation, at the same entrance Reynolds number.
5. There exist two interdependent separation regions, one in each branch.
6. An increase of the mass flow ratio, γ , is due to both an increase of the thickness, τ_D , of the separated region in the main branch and a decrease of the thickness, τ_d , of the separated region in the side branch.
7. Generation, growth and shedding of banana shaped vortices have been observed in the separated region of the main branch. In the

side branch, double-helicoidal flow has been observed.

8. The flow through the bifurcation is highly dependent on the resistance downstream of the bifurcation.

5.2 Suggestions for Further Work

As previously explained, the apparatus has been found inadequate to give valuable quantitative as well as qualitative information for slow flow ($Re \leq 1000$). Reducing the width of the openings at the top of the two constant head tanks will eliminate the surface tension problems which appear at slow flow, and using the visualization technique recommended by D.J. Baker²⁰ for small fluid velocities will be an excellent alternative for the hydrogen bubble technique.

This visualization technique uses a pH indicator. If, on reverse position, (see Figure 3.13), the d.c. voltage applied between the two electrodes is kept sufficiently low, no hydrogen bubbles can be produced at the copper ring. Moreover, if the aqueous solution contains 0.01% of thymolsulphonephthalein, and is titrated to be on the acid (or yellow) side of the end-point ($pH = 8.0$), the production of hydroxyl ions (OH^-) near the wire will change the colour from yellow to blue. Blue rows will then be generated in place of the previous rows of bubbles. Since the thymol blue molecule remains an ion in solution, no buoyancy effects are present. This method requires an illumination by a sodium arc lamp and cannot be used for flows faster than 2"/sec, since the coloured fluid is swept away from the wire more rapidly than it can be formed in visible amounts.

At very slow flow, levelling the two constant head tanks becomes of primary importance due to the principle of connected vessels. Thus, before undergoing any slow flow tests, one has to find a means of being certain that the openings of the two constant head tanks are exactly at the same level.

Not only the study of very slow flow should lead to interesting results but also the study of very fast flow should indicate whether γ goes to zero when the Reynolds number increases, or not. The present apparatus can also be modified for this type of study by enlarging the openings at the top of the two constant head tanks and using larger flowrator meters.

For the range of Reynolds number ($1000 \leq Re \leq 5000$) for which the actual apparatus gives valuable results, four kinds of studies could be undertaken.

1. The testing of other types of bifurcation such as Y-type or curved bifurcations.

2. Obtaining the velocity profiles throughout the bifurcation by measuring the distances separating two rows of bubbles directly on the photographs. This necessitates, in order to avoid any error due to optical distortions, to superimpose two photographs: one of the actual experiment and the other of a scale in the plane of the wire inside the bifurcation filled with water.

3. The influence of the resistance downstream of the bifurcation on the flow through the bifurcation.

4. The study of the frequency and pattern of vortex shedding, as a function of the Reynolds number and the bifurcation geometry.

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APPENDIX A

VISCOSITY OF THE EXPERIMENTAL FLUID

The experimental fluid is water to which a wetting agent and an antifoam have been added. This solution has a viscosity slightly different than water.

The kinematic viscosities of the solution and of the tap-water of the University of Alberta have been measured with a Cannon-Fenske viscometer.* Table A.1 and Figure A.1 give the variations of these viscosities with the temperature.

It appears that the addition of the wetting agent and the antifoam affects the viscosity of the water by, at the most, 1.30%.

When the temperature of the solution varies, its viscosity varies, so does the Reynolds number for a given flow rate. This Reynolds number can be expressed in terms of the total flow rate ($Q_d + Q_D$). Since

$$Q_d + Q_D = u' \pi \frac{D^2}{4} \quad (A.1)$$

the Reynolds number becomes

*Cannon-Fenske-Ostwald Calibrated Viscometer No. 50 J657.

$$Re = \left[\frac{4}{\pi \nu D} \right] (Q_d + Q_D) \quad (A.2)$$

One can write this as

$$Re = a (Q_d + Q_D) \quad (A.3)$$

when the flow rates are measured in liters per minute, or as

$$Re = b (Q_d + Q_D) \quad (A.4)$$

when the measurements are made in U.S. gallons per minute.

Table A.2 and Figures A.2 and A.3 give the variations of a and b with the temperature of the solution.

Table A.1
Kinematic Viscosity (centistokes) as
a Function of Temperature

Temperature °C	°F	Tap-Water	Solution
20	68	1.015	1.002
40	104	0.669	0.662
60	140	0.487	0.482
80	176	0.380	0.376

Table A.2
Variation of a and b with the Temperature

Temperature °C	°F	a	b
		(liters/mn) ⁻¹	(U.S. gpm) ⁻¹
20	68	668	2536
40	104	1012	3838
60	140	1388	5267
80	176	1781	6756

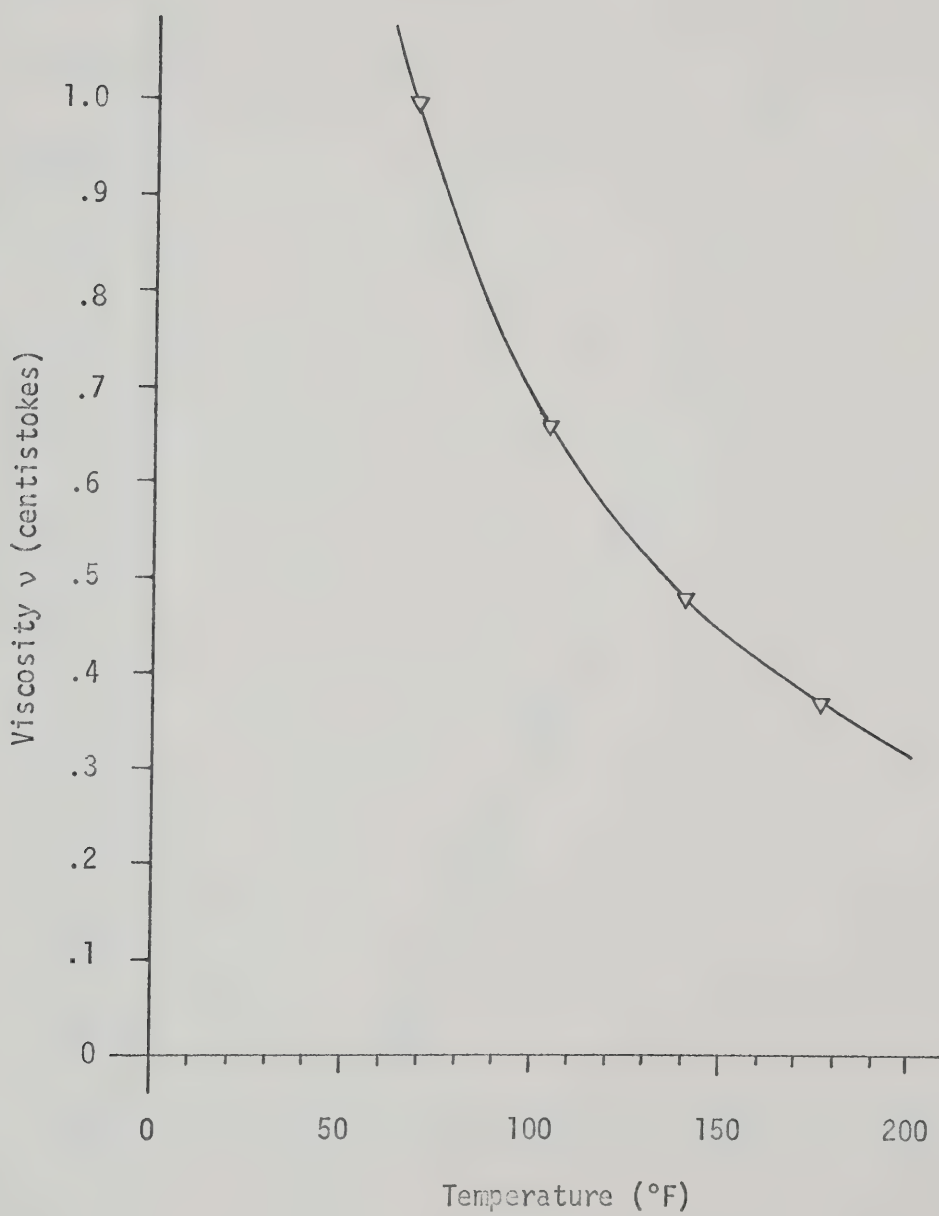


Figure A.1 Kinematic Viscosity of the
Solution versus Temperature

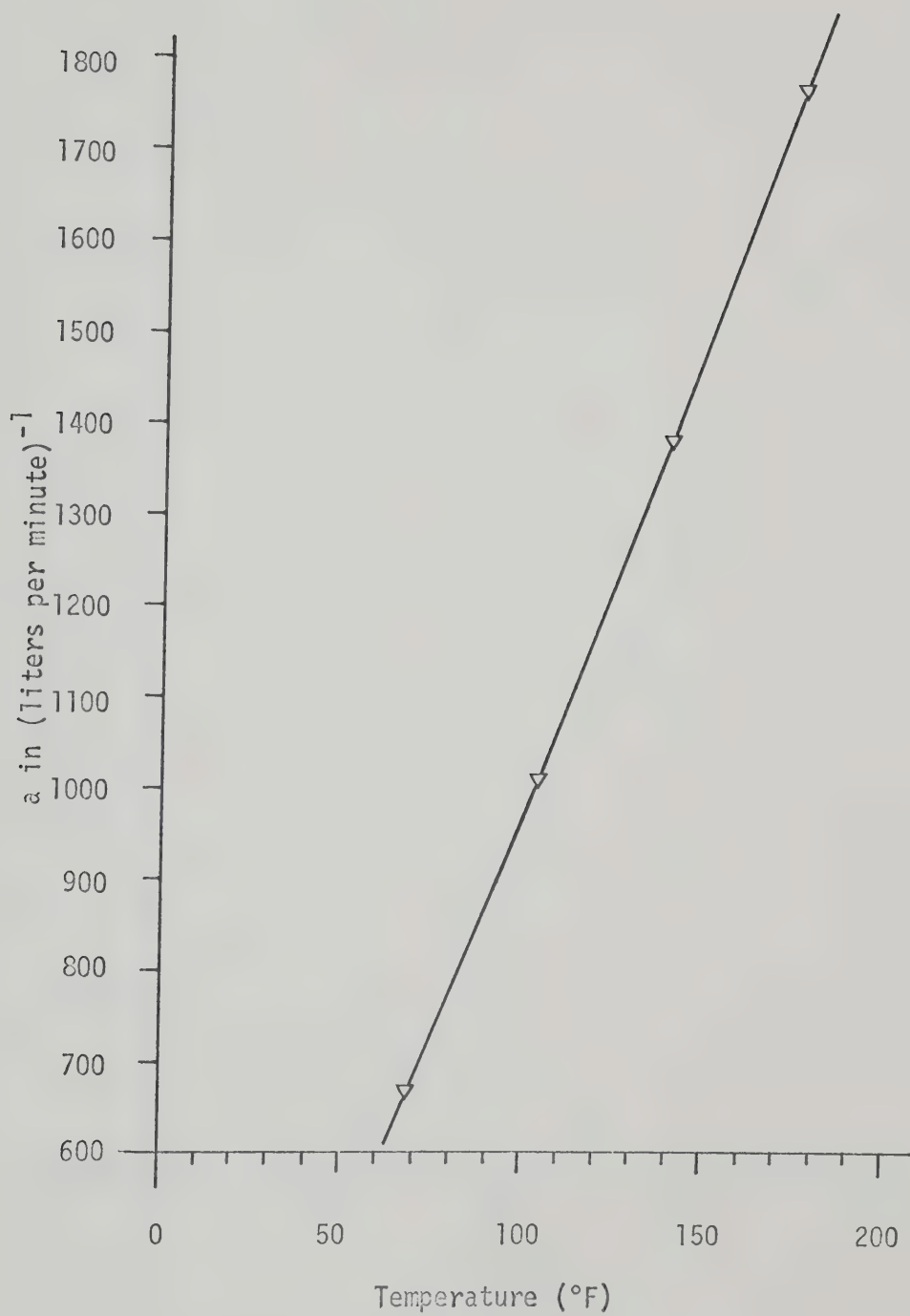


Figure A.2 Value of a versus Temperature

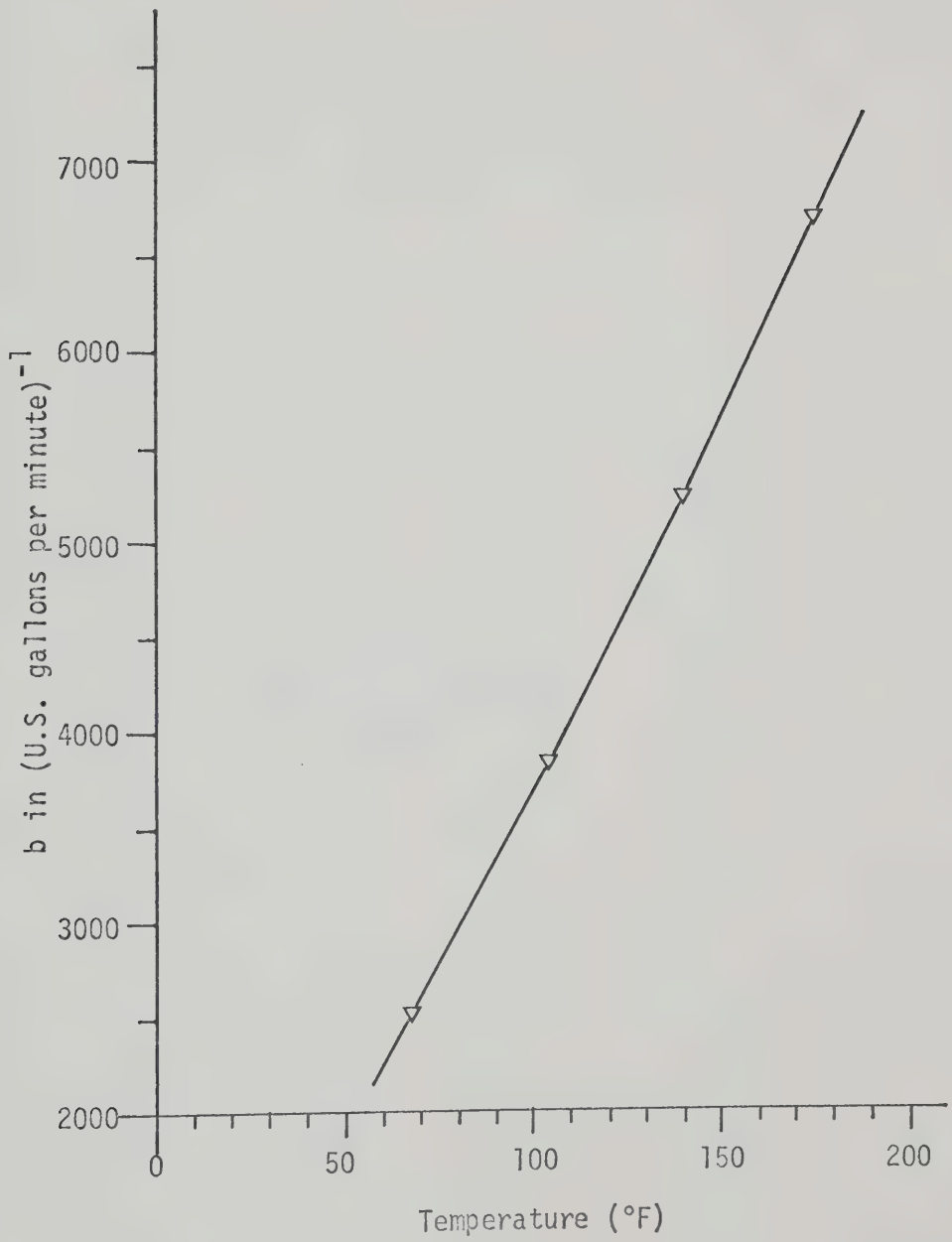


Figure A.3 Value of b versus Temperature

APPENDIX B
TABLES OF RESULTS FOR
CHAPTER IV

Table B.1

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 1.00$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
1020	1.630
1070	1.600
1170	1.535
1220	1.500
1310	1.462
1400	1.410
1590	1.338
1830	1.290
1980	1.272
2140	1.240
2250	1.230
2390	1.216
2550	1.195
2840	1.180

Re	γ
3040	1.158
3150	1.142
3240	1.145
3530	1.105
3720	1.097
3790	1.095
3950	1.080
4180	1.068
4380	1.070
4520	1.065
4720	1.070
4820	1.065
4900	1.055
4980	1.050

Table B.2

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 1$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
940	1.330
950	1.315
1060	1.275
1100	1.265
1250	1.215
1370	1.187
1440	1.175
1530	1.150
1600	1.130
1670	1.120
1800	1.110
1930	1.075
1980	1.070
2120	1.049
2290	1.050

Re	γ
2500	1.045
2750	1.035
2900	1.035
2950	1.033
3150	1.018
3450	1.005
3760	1.008
3850	1.010
3960	0.984
4040	0.975
4360	0.960
4400	0.964
4550	0.953
4870	0.950
5000	0.954

Table B.3

Dependence of γ on Re for

a Sharp Edged Bifurcation with

 $\beta = 1.00$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
650	1.295
730	1.250
780	1.230
860	1.210
1050	1.160
1120	1.135
1230	1.110
1390	1.085
1480	1.060
1660	1.040
1800	1.020
1910	1.015
2140	1.010
2250	1.005
2360	1.000

Re	γ
2550	1.000
2800	1.008
2880	1.000
3120	0.990
3240	0.975
3420	0.960
3640	0.952
3850	0.946
4030	0.942
4150	0.930
4320	0.926
4460	0.912
4600	0.920
4840	0.918
4950	0.918

Table B.4

Dependence of γ on Re for

a Sharp Edged Bifurcation with

 $\beta = 1$ and $\alpha = 1$ ($\theta = 90^\circ$)

Re	γ
770	1.135
840	1.130
900	1.112
1050	1.088
1170	1.078
1265	1.042
1330	1.032
1500	1.020
1620	1.000
1720	0.978
1820	0.972
1950	0.958
2010	0.954
2120	0.954
2220	0.947
2350	0.946
2470	0.940
2540	0.934

Re	γ
2620	0.930
2740	0.940
2850	0.942
3000	0.937
3130	0.930
3250	0.922
3360	0.918
3500	0.918
3670	0.914
3830	0.897
4000	0.888
4100	0.879
4200	0.880
4350	0.876
4530	0.878
4700	0.875
4840	0.868
4950	0.865

Table B.5

Dependence of γ on Re for

a Sharp Edged Bifurcation with

 $\beta = 0.80$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
950	1.630
1030	1.590
1100	1.540
1160	1.498
1220	1.460
1310	1.380
1460	1.315
1520	1.285
1680	1.258
1800	1.230
1930	1.210
2120	1.175
2220	1.155
2390	1.120

Re	γ
2530	1.112
2780	1.092
3010	1.080
3280	1.060
3620	1.045
3900	1.025
4000	1.020
4090	1.015
4240	1.010
4460	1.000
4590	0.990
4750	0.985
4840	0.985
5000	0.984

Table B.6

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.80$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
740	1.360
840	1.328
900	1.280
1070	1.245
1270	1.180
1350	1.150
1470	1.135
1590	1.105
1730	1.068
1840	1.052
1950	1.042
2080	1.030
2260	1.022
2380	1.008
2470	1.007

Re	γ
2600	1.007
2780	1.005
2940	0.990
3150	0.970
3300	0.960
3460	0.954
3620	0.946
3760	0.944
3850	0.944
4070	0.950
4210	0.942
4430	0.930
4650	0.926
4810	0.910
5000	0.903

Re	γ
4600	0.920
4400	0.924
3950	0.950
3700	0.955
3400	0.950
3240	0.956
2880	0.965
2640	0.985
2280	1.000
2040	1.020
1550	1.100
1130	1.215

Table B.7

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.80$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
710	1.150
870	1.135
1070	1.115
1160	1.100
1240	1.070
1330	1.055
1410	1.040
1520	1.020
1710	0.990
1870	0.972
2160	0.952
2270	0.954
2430	0.950
2600	0.951

Re	γ
2780	0.940
2880	0.938
3000	0.934
3150	0.940
3350	0.930
3580	0.920
3850	0.895
4000	0.888
4210	0.885
4400	0.870
4520	0.868
4720	0.868
4910	0.867
5000	0.870

Table B.8

Dependence of γ on Re for

a Sharp Edged Bifurcation with

 $\beta = 0.80$ and $\alpha = 1$ ($\theta = 90^\circ$)

Re	γ
640	1.140
710	1.128
820	1.118
950	1.093
1050	1.078
1180	1.065
1250	1.035
1300	1.025
1380	1.020
1490	1.000
1650	0.976
1730	0.970
1850	0.962
1980	0.948
2060	0.942
2200	0.932
2400	0.918
2520	0.924

Re	γ
2620	0.920
2710	0.910
2900	0.900
3070	0.898
3200	0.890
3290	0.888
3400	0.886
3520	0.888
3700	0.876
3820	0.866
3930	0.870
4040	0.860
4200	0.848
4330	0.844
4500	0.828
4730	0.820
4960	0.816

Table B.9

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.60$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
820	1.500
840	1.485
870	1.455
940	1.360
1020	1.320
1120	1.250
1240	1.210
1460	1.130
1560	1.100
1680	1.075
1770	1.050
1850	1.032
2020	1.000
2210	0.985

Re	γ
2420	0.951
2630	0.934
2890	0.907
3070	0.900
3400	0.882
3720	0.850
3850	0.838
4030	0.830
4140	0.817
4380	0.814
4520	0.809
4670	0.798
4800	0.796
4970	0.786

Table B.10

Dependence of γ on Re for
a Sharp Edged Bifurcation with
 $\beta = 0.60$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
650	1.250
730	1.195
880	1.150
1050	1.090
1200	1.055
1300	1.015
1390	1.002
1540	0.980
1640	0.965
1770	0.938
1890	0.926
2100	0.900
2230	0.896
2420	0.880

Re	γ
2560	0.870
2750	0.871
2930	0.878
3130	0.856
3320	0.840
3600	0.835
3780	0.824
4000	0.815
4100	0.812
4190	0.805
4300	0.800
4490	0.796
4740	0.786
4950	0.775

Table B.11

Dependence of γ on Re for

a Sharp Edged Bifurcation with

 $\beta = 0.60$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
650	1.035
720	1.025
800	1.010
920	1.000
1050	0.992
1120	0.976
1200	0.960
1350	0.946
1490	0.920
1610	0.900
1730	0.898
1880	0.890
2030	0.874
2200	0.867
2340	0.854

Re	γ
2520	0.845
2750	0.848
3000	0.845
3200	0.832
3450	0.817
3700	0.792
3840	0.780
4000	0.778
4200	0.774
4320	0.758
4380	0.754
4500	0.742
4600	0.738
4800	0.725
5000	0.720

Table B.12

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.60$ and $\alpha = 1$ ($\theta = 90^\circ$)

Re	γ
750	1.000
800	0.992
840	0.983
900	0.978
950	0.978
1020	0.967
1070	0.952
1180	0.934
1250	0.924
1300	0.920
1420	0.902
1500	0.884
1630	0.868
1760	0.854
1830	0.840
1900	0.834
2050	0.810

Re	γ
2160	0.804
2380	0.792
2580	0.790
2820	0.790
3000	0.775
3180	0.774
3360	0.752
3450	0.746
3650	0.726
3800	0.728
3960	0.728
4130	0.718
4240	0.710
4340	0.705
4520	0.700
4730	0.684
4900	0.670

Table B.13

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.40$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
680	0.936
760	0.910
810	0.890
920	0.850
1040	0.815
1170	0.778
1310	0.736
1480	0.688
1570	0.661
1670	0.620
1760	0.607
1850	0.583
2000	0.570
2140	0.553

Re	γ
2420	0.535
2640	0.535
2780	0.530
3000	0.514
3180	0.500
3360	0.490
3500	0.487
3660	0.474
3950	0.458
4170	0.447
4360	0.420
4520	0.422
4650	0.413
4850	0.404

Table B.14

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.40$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
740	0.860
800	0.845
910	0.820
980	0.790
1080	0.770
1200	0.750
1350	0.723
1570	0.705
1700	0.690
1770	0.675
1900	0.657
2080	0.624
2300	0.596
2420	0.590

Re	γ
2500	0.580
2780	0.574
2900	0.556
3040	0.548
3220	0.538
3420	0.534
3600	0.518
3790	0.508
3900	0.500
4050	0.500
4320	0.498
4500	0.484
4760	0.474
4940	0.460

Table B.15

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.40$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
760	0.710
890	0.685
1010	0.675
1180	0.654
1320	0.638
1440	0.628
1580	0.605
1660	0.596
1730	0.590
1900	0.576
2000	0.560
2100	0.554
2270	0.540
2450	0.530

Re	γ
2740	0.522
2900	0.509
3130	0.500
3300	0.498
3420	0.490
3570	0.482
3720	0.472
3900	0.466
4020	0.467
4200	0.460
4400	0.452
4620	0.436
4820	0.435
4900	0.434

Table B.16

Dependence of γ on Re for
 a Sharp Edged Bifurcation with
 $\beta = 0.40$ and $\alpha = 1$ ($\theta = 90^\circ$)

Re	γ	Re	γ
720	0.668	2520	0.500
830	0.653	2680	0.487
890	0.643	2820	0.484
1050	0.638	2940	0.480
1150	0.624	3100	0.464
1270	0.612	3250	0.465
1370	0.598	3430	0.448
1450	0.594	3540	0.442
1540	0.586	3730	0.426
1600	0.576	3830	0.422
1720	0.562	3980	0.424
1790	0.556	4100	0.412
1900	0.543	4280	0.402
2000	0.534	4450	0.400
2140	0.521	4680	0.394
2280	0.510	4800	0.396
2380	0.503	5000	0.380

Table B.17

Dependence of γ on Re for

a Round Edged Bifurcation with

 $\beta = 1.00$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
1040	1.740
1190	1.548
1220	1.510
1430	1.400
1640	1.345
1790	1.298
2040	1.253
2150	1.235
2390	1.210
2640	1.190
2860	1.165
3000	1.150
3140	1.138

Re	γ
3250	1.128
3440	1.110
3610	1.100
3780	1.092
3990	1.085
4100	1.075
4230	1.080
4390	1.065
4500	1.055
4620	1.070
4740	1.060
4850	1.058
5000	1.050

Re	γ
1080	1.600
1150	1.518
1450	1.405
1840	1.280
2010	1.250
2370	1.187
2940	1.140
3540	1.097
4300	1.047
4700	1.040

Table B.18

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 1.00$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
900	1.430
1020	1.325
1250	1.240
1410	1.195
1470	1.170
1600	1.152
1720	1.132
1860	1.100
1970	1.092
2200	1.070
2360	1.060
2470	1.062
2600	1.060
2840	1.055

Re	γ
2980	1.040
3180	1.030
3340	1.010
3500	1.000
3690	0.992
3900	1.000
4020	0.988
4150	0.980
4250	0.965
4350	0.960
4560	0.950
4650	0.950
4900	0.948
5000	0.950

Table B.19

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 1.00$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
680	1.225
810	1.210
990	1.155
1190	1.120
1310	1.100
1430	1.075
1530	1.055
1620	1.030
1750	1.010
1850	0.994
1990	0.992
2080	0.978
2200	0.970
2390	0.964

Re	γ
2580	0.964
2680	0.962
2820	0.955
3000	0.948
3150	0.945
3430	0.944
3650	0.936
3920	0.935
4100	0.930
4280	0.930
4420	0.930
4570	0.924
4720	0.908
5000	0.904

Table B.20
 Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 1.00$ and $\alpha = 1.00$ ($\theta = 90^\circ$)

Re	γ
670	1.192
850	1.160
1000	1.125
1170	1.085
1270	1.060
1430	1.035
1530	1.020
1740	0.994
1870	0.975
1960	0.974
2150	0.960
2280	0.950
2480	0.952
2590	0.950

Re	γ
2750	0.946
2960	0.934
3060	0.920
3170	0.918
3310	0.916
3480	0.914
3750	0.908
3940	0.906
4130	0.892
4340	0.890
4430	0.890
4560	0.882
4740	0.870
4950	0.865

Table B.21

Dependence of γ on Re for

a Round Edged Bifurcation with

 $\beta = 0.80$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
840	1.790
990	1.650
1070	1.520
1270	1.435
1460	1.362
1630	1.315
1760	1.283
1930	1.236
2110	1.190
2350	1.156
2500	1.135
2620	1.123
2750	1.112

Re	γ
2830	1.106
2920	1.100
3000	1.095
3250	1.082
3400	1.065
3540	1.060
3790	1.052
4000	1.040
4220	1.032
4360	1.015
4600	1.010
4910	1.002
5000	1.000

Table B.22

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.80$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
830	1.380
930	1.350
1150	1.245
1260	1.212
1370	1.180
1520	1.145
1680	1.115
1800	1.080
2030	1.061
2180	1.042
2310	1.030
2420	1.015
2600	1.007
2720	1.008

Re	γ
2910	1.000
3050	0.995
3180	0.990
3310	0.978
3470	0.957
3550	0.965
3640	0.966
3830	0.964
3950	0.960
4150	0.950
4300	0.935
4460	0.928
4680	0.930
4840	0.932

Table B.23

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.80$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
680	1.162
830	1.150
1000	1.130
1120	1.105
1240	1.095
1320	1.060
1430	1.035
1590	1.018
1730	1.000
1840	0.980
1970	0.972
2140	0.950
2350	0.950
2490	0.940
2620	0.940

Re	γ
2790	0.937
2930	0.930
3150	0.920
3300	0.917
3490	0.912
3740	0.910
3880	0.915
4140	0.910
4280	0.895
4360	0.897
4430	0.890
4550	0.895
4680	0.882
4850	0.875
4960	0.867

Table B.24

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.80$ and $\alpha = 1$ ($\theta = 90^\circ$)

Re	γ
700	1.150
870	1.138
970	1.125
1160	1.090
1220	1.075
1350	1.040
1440	1.020
1530	1.000
1670	0.984
1800	0.964
2010	0.940
2140	0.928
2230	0.932
2380	0.927
2580	0.929

Re	γ
2720	0.931
3000	0.926
3090	0.920
3260	0.904
3470	0.887
3550	0.868
3760	0.840
3910	0.830
4050	0.824
4170	0.820
4310	0.821
4550	0.824
4650	0.815
4800	0.810
4950	0.808

Table B.25

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.60$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
750	1.720
820	1.530
950	1.445
1090	1.342
1160	1.295
1230	1.272
1310	1.225
1440	1.180
1490	1.160
1620	1.132
1670	1.123
1850	1.082
2010	1.050
2120	1.020

Re	γ
2280	1.000
2410	0.980
2600	0.950
2740	0.930
3040	0.904
3300	0.870
3500	0.862
3740	0.848
3880	0.843
4190	0.834
4420	0.825
4600	0.816
4820	0.800
5000	0.794

Table B.26

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.60$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
830	1.250
860	1.240
900	1.190
970	1.160
1010	1.148
1080	1.130
1160	1.115
1220	1.102
1270	1.090
1360	1.065
1470	1.035
1600	1.000
1730	0.965
1980	0.934

Re	γ
2090	0.925
2200	0.918
2470	0.905
2610	0.906
2900	0.880
3250	0.840
3380	0.828
3550	0.832
3740	0.824
4090	0.805
4270	0.795
4500	0.796
4740	0.789
4950	0.780

Table B.27

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.60$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
670	1.075
790	1.052
930	1.025
1050	1.000
1190	0.982
1260	0.965
1330	0.953
1500	0.934
1610	0.924
1780	0.900
1920	0.885
2050	0.865
2220	0.850
2370	0.840

Re	γ
2530	0.830
2710	0.832
2920	0.830
3100	0.820
3250	0.812
3370	0.800
3660	0.788
3840	0.790
4070	0.785
4200	0.783
4380	0.780
4550	0.760
4860	0.745
5000	0.736

Table B.28

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.60$ and $\alpha = 1$ ($\theta = 90^\circ$)

Re	γ
610	1.075
820	1.020
900	1.000
1030	0.982
1090	0.968
1210	0.954
1370	0.932
1520	0.904
1680	0.878
1750	0.865
1920	0.830
2140	0.816
2330	0.800
2460	0.795

Re	γ
2660	0.787
2800	0.782
2950	0.775
3090	0.772
3290	0.760
3500	0.755
3610	0.753
3790	0.730
4000	0.718
4100	0.715
4270	0.720
4520	0.705
4720	0.698
4910	0.690

Table B.29

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.40$ and $\alpha = 1/3$ ($\theta = 30^\circ$)

Re	γ
460	1.200
680	0.965
780	0.935
830	0.925
1000	0.848
1070	0.814
1190	0.776
1300	0.750
1460	0.704
1730	0.654
1900	0.625
2000	0.604
2140	0.586
2230	0.573

Re	γ
2400	0.568
2580	0.550
2740	0.535
2840	0.530
3130	0.506
3340	0.493
3490	0.485
3650	0.480
4030	0.454
4230	0.450
4320	0.448
4500	0.445
4690	0.435
5000	0.413

Table B.30

Dependence of γ on Re for

a Round Edged Bifurcation with

 $\beta = 0.40$ and $\alpha = 5/9$ ($\theta = 50^\circ$)

Re	γ
640	0.950
680	0.935
790	0.890
910	0.863
1060	0.826
1200	0.780
1310	0.762
1460	0.727
1590	0.688
1730	0.670
1870	0.650
2010	0.630
2210	0.612
2320	0.598

Re	γ
2450	0.590
2540	0.574
2660	0.572
2800	0.565
2940	0.550
3190	0.540
3450	0.520
3740	0.500
3890	0.500
4070	0.500
4230	0.495
4600	0.477
4780	0.460
5000	0.450

Table B.31

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.40$ and $\alpha = 7/9$ ($\theta = 70^\circ$)

Re	γ
650	0.760
700	0.756
800	0.745
950	0.725
1060	0.716
1160	0.700
1200	0.692
1310	0.675
1390	0.660
1620	0.625
1760	0.600
1840	0.592
2010	0.575
2170	0.554

Re	γ
2300	0.546
2430	0.544
2540	0.530
2680	0.523
2820	0.518
3000	0.507
3250	0.500
3450	0.488
3750	0.476
3890	0.476
4170	0.460
4350	0.450
4680	0.435
4970	0.440

Table B.32

Dependence of γ on Re for
 a Round Edged Bifurcation with
 $\beta = 0.40$ and $\alpha = 1$ ($\theta = 90^\circ$)

Re	γ
620	0.723
800	0.681
970	0.673
1080	0.657
1200	0.634
1340	0.610
1500	0.592
1660	0.570
1810	0.558
1980	0.542
2070	0.537
2200	0.520
2320	0.514
2470	0.500
2590	0.502

Re	γ
2690	0.497
2810	0.500
2930	0.488
3070	0.486
3180	0.474
3230	0.470
3360	0.470
3470	0.464
3650	0.460
3800	0.444
3930	0.439
4100	0.434
4350	0.428
4640	0.415
4900	0.406

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